A STUDY ON GAME THEORY

DISSERTATION SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE AWARD OF BACHELOR'S DEGREE IN MATHEMATICS

Jointly

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DECLARATION

We hereby declare that this project entitled 'A STUDY ON GAME THEORY' is a bonafide record of work done by me under the guidance of Dr. Joby Mackolil, Assistant professor, Department of Mathematics, Bharata Mata College, Thrikkakara and this work has not previously formed by the basis for the award of any academic qualification, fellowship or other similar title of any other University or Board.

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CERTIFICATE

This is to certify that the project entitled A STUDY ON GAME THEORY submitted jointly by Athira A S, Rizvana M A, Edwin Jolly in partial fulfillment requirement of Under Graduate Degree in Mathematics, is a bonafide record of the studies undertaken them under my supervision at the Department of Mathematics, Bharata Mata College, Thrikkakara during 2021-24

This dissertation has not been submitted for any other degree elsewhere.

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CHAPTER 1:
INTRODUCTION

A mathematical theory that addresses the fundamental elements of competitive environments is called the games theory. When multiple people or groups are trying to make decisions with competing goals, this notion might be useful. In these kinds of situations, a choice made by one decision maker influences the decisions of one or more other decision makers. This idea can be used in a wide range of scenarios, including contests between candidates for office, rival groups strategizing for war, and more. It is founded on Von Neumann's minimax principle, which states that every competitor will behave in a way that minimizes his maximum loss or maximizes his minimum gain. Thus far, only basic competitive issues have examined by means of this mathematical theory. Just the process and guidelines for choosing plays are described in the theory; it does not address the optimal way to play a game.

1.1 Characteristics of games

A competitive game has the following characteristics:

- The number of competitors or participants is limited. The game is known as the Two Person Game when there are just two players. An n-person game is one where n is more than 2.
- A list of the limited number of options available to each participant is provided. Each participant's list could differ.
- Every participant is aware of every option open to them, but they are unsure of which option they will ultimately select.
- When every player selects one of the options given to him, a play is considered to have taken place. It is assumed that the decisions are made concurrently, meaning that each participant is in the dark about the options until he makes his own.
- Each combination of steps generates a result that benefits all parties involved. The gain could be zero, negative, or positive. We refer to a negative gain as a loss.
- A participant's gain is based on both his own and other people's activities.

1.2 Basic Concepts of Game Theory

- **Players:** Decision-making in the game is done by entities or people.
- **Strategies:** Plans of action that any player can access

- **Pay-off:** The incentives or results that players obtain from their decisions during a strategic encounter are referred to as pay-offs.
- **Gain:** The term "gain" describes the advantage or profit that a player gets from a specific course of play or tactic.
- **Loss:** The term "loss" describes the unfavorable result or disadvantage that a person receives in a game as a result of their decisions or other players' actions.

1.3 Mathematics and Games

Games and mathematics have an interesting relationship. A subfield of mathematics called "game theory" studies how players make strategic decisions in games. It aids in the analysis and comprehension of the methods and results of a variety of games, from easy ones like Rock, Paper, Scissors to difficult strategic ones like Chess or Poker.

Payoff matrices, Nash equilibria, and optimal tactics are only a few of the concepts and methods available in mathematics for modeling and analyzing games. We may better comprehend the optimal tactics and results in various game settings thanks to these mathematical tools.

Games and mathematics go hand in hand because they both enable us to understand the strategic nuances of a variety of games and real-world scenarios. This intriguing topic blends strategic thinking with logical reasoning.

Table that Illustrates the outcomes of Rock-Paper-Scissors Game:

Player 2 / Player 1	Rock	Paper	Scissors
Rock	**Tie**	**Player 2 wins**	**Player 1 wins**
Paper	**Player 1 wins**	**Tie**	**Player 2 wins**
Scissors	**Player 2 wins**	**Player 1 wins**	**Tie**

Each cell in the table above represents how the game will play out based on the choices made by players 1 and 2. When both players choose the same option, it results in a "tie," which indicates that no one wins. The other cells indicate which player wins based on possible combo combinations.

1.4 Strategy

A strategy in game theory is a comprehensive course of action that a player choose to accomplish their goals in a specific game or situation involving decision-making. At each choice point, it outlines the player's course of action, taking into account the potential actions of other players and their expected reactions.

In game theory, strategies can take several forms:

1. **Pure Strategy:** A pure strategy is a determined action or decision made by a player. It entails picking one specific option from the list of options.

Eg: Let's look at a simplified version of the game, in which "Up" (U) and "Down" (D) are the only two pure strategies available to players A and B. The payoff matrix can resemble this:

		Player B	
		U	D
Player A	U	3, 3	0, 2
	D	2, 0	1, 1

In this matrix:

- Player A selects between rows labeled "Up" and "Down" in this matrix.
- The option that Player B choose is "Up" or "Down" (columns).
- The payouts to Player A and Player B are indicated by the numbers in each cell.

2. **Mixed Strategy:** In a mixed strategy, various pure strategies are alternated at random based on a probability distribution. The player choose every action that has a chance of happening rather than settling on just one.

Eg: Consider the following scenario: Player A is presented with two pure strategies, "Heads" (H) and "Tails" (T), with probabilities of p and (1-p), respectively.Player B makes comparable decisions. The payoff matrix may resemble this

Player B

Player A	H	T
H	1, 2	3, 0
T	0, 1	2, 3

The probabilities of selecting "Heads" for Player A and Player B are denoted by p and q, respectively.

3. **Deterministic Strategy:** A deterministic strategy is a pure strategy in which there is no unpredictability and the player's actions are fully planned at every decision point.

Eg: Considering a game where Player A and Player B each have two pure strategies, "Left" (L) and "Right" (R), we can examine a deterministic strategy. In spite of the opponent's decision, both players consistently select the same course of action. The payoff matrix may resemble this:

Player B

Player A	L	R
L	2, 1	0, 3
R	1, 2	3, 0

Here, there is no randomness in either Player A's or Player B's choice of "Left" or "Right″.

4. **Stagnant Strategy:** In a stagnant strategy, changes are made gradually or repeatedly. It remains unchanged in light of previous deeds or results.

Eg: Both players in this game have the option of using "High" (H) or "Low" (L) strategies. This is how the payout matrix could appear:

		Player B	
		H	**L**
Player A	**H**	**2, 2**	**1, 1**
	L	**1, 1**	**3, 3**

In this matrix:

- Both "High" (H) and "Low" (L) are the two strategies that Player A and Player B have.
- The payoffs to Player A and Player B are represented by the numbers in each cell, correspondingly.
- If both players select "High," they will split the payout of two.
- In the event that both players select "Low," they will each be paid three.
- If two players select "High" and "Low," then the player who selects "High" will receive 1 and the player who selects "Low" will receive 1 as well.

In a stationary strategy, regardless of past results, both players always select the same strategy (either "High" or "Low") in every round. This is not a strategy that adapts to the opponent's moves or changes over time.

5. **Trigger Strategy:** In a trigger strategy, desertion is punished by the other player defecting in return, while cooperation is only allowed as long as the other player cooperates.

Eg: Here, we'll examine a repeated Prisoner's Dilemma scenario in which both players have the option to "cooperate" (C) or "defect" (D). The possible payout matrix is:

		Player B	
		C	**D**
Player A	**C**	**3, 3**	**0, 5**
	D	**5, 0**	**1, 1**

In this case, both players earn a payout of 3 if they collaborate (C). In the event when one player cooperates while the other defects (D), the cooperator receives 0 and the defector receives 5. They each get paid one if they both make a flaw.

6. **Tit-for-Tat Strategy:** This is a particular kind of strategy that is frequently employed in games that are played repeatedly, in which a player cooperates at first and then imitates the opponent's prior move in later rounds.

Eg: Consider two businesses, A and B, are debating whether or not to make an advertising investment (A) (N). Their earnings are contingent upon both their own and their rival's decisions. This is the matrix of payoffs:

		Company B	
		A	N
Company A	A	3, 3	0, 5
	N	5, 0	1, 1

In this matrix:

- Both businesses receive a payback of three if they invest in advertising (A) because of the enhanced market exposure.
- If Company B does not invest in advertising (N) and Company A does (A), Company A will receive a payoff of 0 (because their investment did not yield a competitive advantage) and Company B will receive a payoff of 5 (since they will be able to save money on advertising and possibly increase their market share).
- The scenario is inverted, with Company A receiving a payout of 5 and Company B receiving a payout of 0, if Company A does not invest (N) while Company B invests (A).
- Both businesses receive a minor payout of 1 if they decide not to invest (N), as they save money on advertising but might lose out on market opportunities.

Let's now use the Tit-for-Tat tactic to address this situation:

- Initially, neither of the two businesses spends (N) on advertising.
- In the next round, every business imitates what its rival did before. In the following round, the corporation will also invest (A) if the competition does. The corporation refrains from investing (N) in the event that its rival does not.

In this case, the Tit-for-Tat strategy not only offers a way to match competitive actions by both companies, but it also promotes cooperation when they choose not to invest. This may result in a stable equilibrium where both businesses spend money on advertising, increasing their exposure to the market and saving money when their rivals don't.

CHAPTER 2:
TYPES OF GAMES

2.1 Static and Dynamic games

2.1.1 Static Games:

Static games are those in which every player make a decision at the same time without being aware of what the other players have decided. Based on their personal tastes and perceptions of the methods of the other players, each player choose their own approach. The combination of strategies selected by each player determines the game's outcome. It's as if everyone is moving simultaneously and not knowing what the others will do. It's a thought-provoking idea for comprehending strategic interactions.

Eg: Alice and Bob

Alice and Bob, two suspects, are taken into custody in connection with a criminal incident. The police make a bargain with each of them: if they both remain silent and the other confesses, the person who confesses will get a less term while the other will face a heavier penalty. They will each receive a modest punishment if they remain silent. However, if they both confess, they will receive a quite severe punishment. Both Alice and Bob must make decisions in this game without knowing what the other will decide. It all comes down to balancing their personal interests against what might happen if the other player makes a different choice.

2.1.2 Dynamic Games:

Dynamic games are those in which players make decisions in a sequential fashion while keeping in mind the strategies and actions of earlier players. When playing dynamic games, players can see what other players do before making their own moves, which can have a big impact on how they decide what to do. Players must think about how their choices may affect other players' future plays, which gives the game an extra level of strategy and suspense. It's similar to a game of chess in that every move you make has an impact on later moves and results.

Eg: Chess

Chess is a game that exemplifies dynamics. Players take turns making moves in chess, and every move has an impact on the game's potential outcomes. The players need to use strategic thinking, weighing the possible outcomes of their actions and projecting their opponent's countermoves. As the game progresses, players adjust their strategies in response to shifting board positions and other players' moves. It's a dynamic, intricate game that demands strategic thinking and preparation.

2.2 Cooperative and Non-cooperative Games

2.2.1 Cooperative Games:

Cooperative games are a particular kind of game where participants can create alliances and cooperate to achieve a common objective. Players in these games can choose to work together, communicate, and form legally binding agreements. The main thesis is that teamwork is superior to individual action in terms of achieving better results for players. In cooperative games, participants frequently have to coordinate, negotiate, and divide rewards. It's similar to working together with pals to do a task or solve a puzzle. It all comes down to maximizing the advantages as a team.

Eg: Prisoners dilemma

Two suspects, are taken into custody on suspicion of a crime. They are put in different rooms and given the choice to work together or turn on one another. Player-1 and Player-2 will both get a moderate sentence if they decide to cooperate and keep quiet. But if one decides to turn on the other and confesses while the other remains silent, the one who turns traitor gets a less sentence while the other gets a heavier one. Should they decide to turn against one another, they will both receive a quite severe punishment. The conflict between a person's self-interest and the advantages of teamwork is brought to light in the game.

2.2.2 Non-cooperative Games:

Non-cooperative games are those in which participants decide on their own, without consulting one another or reaching formal agreements. Each player in these games acts only in their own best interests, not thinking about how their decisions may affect other players. The decisions made by each player alone will decide how the game turns out. Games that aren't cooperative frequently entail rivalry, strategic planning, and the desire for individual advantage. It resembles a game where everyone is competing with one another to win.

Eg: Rock-Paper-Scissors

In this easy-to-learn game, two players take turns selecting one of the three options: rock, paper, or scissors. The way the three options interact—rock defeats scissors, scissors beats paper, and paper beats rock—determines the game's conclusion. By selecting an option that can override their opponent's selection, each player attempts to outsmart and anticipate the other. The object of the game is to outwit your opponent by using strategy and fast thinking.

2.3 Zero-sum and Non-zero-sum Games

2.3.1 Zero-sum Games:

A game is said to be zero-sum if all of the participants' combined wins and losses equal zero. Put otherwise, for every point that a player makes, another player loses the same amount. Similar to a fixed pie, every win for one player means losses for others. Competitive sports like chess and tennis are examples of zero-sum games, in which the win of one player immediately correlates to the loss of the other. The objective of these games is to maximize one's own gain while minimize that of the opponent, with the total reward remaining constant. It's all about attempting to win and using smart thinking.

Eg: Poker

Poker is a prime example of a zero-sum game. In poker, participants fight it out for a common pot of money. Throughout the entire game, the total amount of money in the pot stays the same. The other players lose an equal amount when a player wins a hand and takes home a share of the pot. In poker, the total of all wins and losses is always zero. In order to win, players must outsmart and outplay their rivals in this game of skill, strategy, and bluffing.

2.3.2 Non-zero sum Games:

A game is considered non-zero sum if the cumulative gains and losses of all participating players do not equal zero. Stated differently, it is feasible for any player to attain favorable results and so enhance their overall payout. Non-zero sum games allow for the possibility of mutual benefits, in contrast to zero-sum games where one player's gain is directly canceled by another player's loss. This category frequently includes cooperative games like partnerships and negotiations. Players can cooperate to accomplish goals that benefit all parties involved. It all comes down to identifying win-win situations and encouraging collaboration.

Eg: Commercial Partnership

The commercial partnership as an illustration of a non-zero sum game. When two businesses join together, they can cooperate and strive toward shared goals. Both businesses stand to gain from pooling their resources, knowledge, and networks in order to expand and raise their earnings. In this case, one company's success does not always imply the other's failure. Instead, both businesses can prosper and provide a win-win result by working together and creating synergy. It's a fantastic method to capitalize on one another advantages and accomplish success together.

2.4 Symmetric and Asymmetric Games

2.4.1 Symmetric Games:

Symmetric games are those in which every player faces the same payout structure and has access to the same set of strategies. It basically indicates that every player has the same options and outcomes and is in the same situation. Players in a symmetric game are identical to one another in terms of their roles, skills, and preferences.

Eg: War

Each player has a deck of cards, and they alternately expose the top card in the deck to the other player. The round is won and both cards are taken by the person with the higher-ranked card. In the event that all the cards are of the same rank, there is a tie, and the players proceed to "war," in which they turn over more cards one by one before revealing the next. All of the cards on the table are won by the player whose card has the better ranking. To gather every card in the deck is the aim. It's a symmetric game because both players have the same deck of cards and can have the same results.

2.4.2 Asymmetric Games:

Asymmetric games are those in which players must contend with disparate payout structures or diverse sets of possible options. It's similar to playing a game where every participant has different powers, choices, or objectives. For instance, the objective of a game of chess is to checkmate the opponent's king. Each player has a unique set of pieces with unique powers. There is an asymmetry in the game because no two players have the same strategy or possible outcomes. Players in asymmetric games frequently need to modify their tactics in light of each game's particular benefits and drawbacks. It makes the game more intricate and requires more strategic thought.

Eg: Hide and Seek

In this game, the other players hide while one player assumes the role of the seeker. While the hiders want to remain hidden until the finish of the game, the seeker's objective is to locate and tag the concealed players. There is an imbalance in the game due to the players' differing roles and strategy. The hiders must locate cunning hiding places and evade discovery, while the seekers must search and navigate the playing area. Each player in this exciting and entertaining game has a distinct function and goal.

CHAPTER 3
NASH EQUILIBRIUM

The term "Nash equilibrium" in game theory refers to a stable situation in which no party has an incentive to alter their strategy. It bears the name of John Nash, the mathematician who first proposed this concept. Consider a game where there are two players and a variety of strategies available to them. When both players have selected their strategies and neither can gain by altering their approach on their own, a Nash equilibrium is reached. It resembles a moment of equilibrium where all players are trying their hardest given what the other players are doing.

Examine the basic illustration of the Prisoner's Dilemma. Two of the criminals in this scenario are taken into custody and questioned independently. Each can choose to disclose their accomplice (by confessing) or assist the authorities (by remaining silent). The decisions taken by both inmates determine the results, which are symbolized by prison sentences.

When both prisoners confess in the Nash equilibrium of the Prisoner's Dilemma, the result is less than ideal for both sides. Even though working together could result in a better overall result, each prisoner's self-interested choice ultimately has a negative effect on the other. Since any divergence will only harm their individual outcome, neither prisoner has a motivation to change their tactics in this situation.

Nash equilibrium extends beyond the Prisoner's Dilemma to a variety of strategic interactions, such as corporate competition, political discussions, and auctions. Participants in any situation have to take into account not only their own goals and preferences, but also other people's possible reactions.

It's crucial to remember that Nash equilibrium does not ensure that every participant will have the greatest possible outcome. The equilibrium that is attained occasionally might not be desirable or inefficient from a group perspective. This emphasizes the drawbacks of making decisions only out of self-interest and emphasizes the advantages of collaboration and coordination.

Eg 1: Prisoner's Dilemma

Assume Alice and Bob, two suspects who have been taken into custody in connection with a criminal investigation. Although there isn't enough evidence to prove them guilty of the primary allegation, the police believe they committed a lesser crime. After separating Alice and Bob, the cops make them an offer:

- Alice faces a one-year prison while Bob faces a five-year sentence if she remains silent and Bob confesses.
- Bob and Alice will each receive a two-year sentence if they remain silent.
- Bob and Alice will each receive a three-year sentence if they both confess.

This is where the problem arises. Bob and Alice must decide together without knowing each other's plans. Alice faces a stiffer punishment should Bob confess if she remains silent. However, if Bob remains silent and Alice confesses, she too faces a lengthier term. Bob is in the same boat.

The Nash equilibrium in this case occurs when Bob and Alice both confess. This is due to the fact that, regardless of what the other person does, confessing to oneself yields the finest results. But their combined result would be better if they both remained silent.

Therefore, the Prisoner's Dilemma illustrates how people might, while acting in their own self-interest, result in a less than ideal outcome when they choose not to collaborate. It's an intriguing idea in game theory

Eg 2: Stoplight Game

Simplifying strategic interaction, the Stoplight game is frequently used to demonstrate the ideas of Nash equilibrium and the prisoner's dilemma. At a stoplight in this game, two players—let's call them Player A and Player B—must choose:

- **Go or stop decision:** Each participant must choose for themselves whether to "go" or "stop" at the stoplight.
- **Payout Structure:** The rewards are determined by the collective choices made by both players. Should both players decide to "go," a collision will result in a negative payout for each of them. In the event that both players decide to "stop," they will each get a little positive payout to offset the waiting costs. Should one player decide to "go" and the other to "stop," the player who goes will receive a significant positive payment, symbolizing a speedy arrival at their destination, while the player who stops will receive nothing.
- **Strategic Interaction:** When making a choice, each player must take the other player's possible course of action into account. They anticipate what their opponent will do and try to maximize their own reward.
- **Nash equilibrium:** When every player's approach is the best one given the other player's strategy, there is a Nash equilibrium. The Nash equilibrium in the Stoplight game usually occurs when both players decide to "go," which leads to a collision and a negative payout for both of them.

- An analogy of the prisoner's dilemma to the Stoplight game is that, like two suspects in a crime, each player has an incentive to defect (go) regardless of the other player's actions, even though both would benefit if they both cooperated (stopped). This is why the two games are frequently compared.

Table Illustration

Player A		Player B: Go	Player B: Stop
	Go	**+10, +10**	**-1, 0**
	Stop	**0, -1**	**+1, +1**

Within the table:

- The payment to Player A is represented by the first value in each cell.
- The reward for Player B is shown by the second value in each cell.

For instance, both participants earn a +1 payout if they both stop. In the event that Player A moves forward and Player B stops, Player A will be paid -1 for the collision and Player B will be paid 0 (for avoiding the collision). Additionally, if both players move, they will each get paid +10 for moving fast to their destination and -1 for colliding.

Eg 3: Business advertisement game

Company A and Company B are the two players in this game. Both businesses compete in the same market and use advertising to draw clients. Every business must choose how much money to spend on advertising. They have the option of making large, moderate, or small investments.

- **Payoff Structure:** The payoffs are determined by the market share or profit that both companies achieve as a result of their advertising decisions. Both businesses might draw in more clients if they spend a lot on advertising, but their earnings might suffer as a result. In the event that one company makes significant investments and the other makes smaller ones, the company making the larger investments stands to lose out on market share. If both businesses make little investment, they might be able to keep steady earnings but run the danger of losing market share to rivals who engage in more aggressive advertising.

- **Strategic Interaction:** When deciding what to advertise, a business must take into account the possible moves of its rival. Their goal is to increase their profit margin or market share while keeping an eye on their rivals' moves.
  Nash Equilibrium: When one company's advertising plan is the best given the other company's approach, there is a Nash equilibrium. Depending on the precise reward structure and strategic factors, it could entail both companies investing extensively, both investing minimally, or one investing strongly and the other minimally.
- **Nash Equilibrium:** When one company's advertising plan is the best given the other company's approach, there is a Nash equilibrium. Depending on the precise reward structure and strategic factors, it could entail both companies investing extensively, both investing minimally, or one investing strongly and the other minimally.
- **Game Dynamics:** Based on consumer behavior shifts, market input, and competitive actions from other businesses, the companies may modify their advertising techniques over time. Profitability and market share may fluctuate as a result.

Table Illustration

		Company B		
		Heavy	**Moderate**	**Minimal**
Company B	**Heavy**	**-10, -10**	**-5, -15**	**0, -20**
	Moderate	**-15, 5**	**0, 0**	**-10, -10**
	Minimal	**-20, 0**	**-10, -10**	**-5, -5**

This table contains:

- The payout to Company A is represented by the first value in each cell.

- The payout to Company B is shown by the second value in each cell.

For instance, if Business A spends a lot of money on advertising and Business B spends less, Business A might increase its market share and earn a payout of -5, while Company B might lose some market share and earn a payoff of -15.

The table's cells each depict a distinct combination of Company A's and Company B's chosen advertising tactics, along with the associated payouts to each business depending on those tactics.

3.1 Games with Infinite Strategy Spaces

Infinite strategy spaces are examined and tested for a number of characteristics, including stability, uniqueness, and existence of equilibrium.

- **Infinite Strategy Spaces:** Instead of having a limited number of options, participants in these games can choose from a continuum of strategies. Players may be required to make selections in real time or choose from an endless array of options, such as selecting any real number within a predetermined window of time.
- **Equilibrium Analysis:** Finding equilibrium solutions is one of the main goals of equilibrium analysis for games with infinite strategy spaces. For example, a fundamental idea in game theory is the Nash equilibrium, which states that no party has an incentive to unilaterally change their preferred course of action. Determining if equilibria exist and, if so, describing their attributes becomes a central focus in games with infinite strategy spaces.
- **Existence and Uniqueness of Equilibria:** Proving the satisfaction of specific constraints, including strategy space continuity and compactness, is a common task for existence proofs. Conversely, uniqueness necessitates demonstrating that there is a single equilibrium point or a single collection of equilibrium points.
- **Analysis of Stability:** Analyzing equilibrium solutions' stability is crucial once they have been found. When players start with close strategies and their strategies gradually converge to the equilibrium point, this is known as a stable equilibrium. Examining the characteristics of the payoff functions and figuring out whether slight departures from equilibrium methods result in adjustments back towards the equilibrium are standard steps in stability analysis.
- **Comparative Statics:** Knowing how equilibrium solutions alter in reaction to modifications in the game's parameters is another facet of testing games with infinite strategy spaces. Studying how equilibrium tactics change in response to changes in preferences, costs, or other external variables is

one way to do this. Analysis of comparative statics sheds light on how sensitive equilibrium results are to modifications in the underlying environment.

- **Computer Methods:** Complex mathematical tools and computer approaches are typically needed for the analysis of games with infinite strategy spaces. In complicated games where analytical solutions ½½ are not possible, equilibrium solutions are frequently approximated by numerical optimization, simulation techniques, and fixed-point algorithms.

Eg: Matching Pennies Game

Two players, A and B, simultaneously choose to show either heads or tails by placing a coin face down. If the coins match (both heads or both tails), Player A wins $1 from Player B. If the coins don't match, Player B wins $1 from Player A.

- Each player's strategy set consists of two choices: heads or tails.
- The payoffs are symmetric: Player A's payoff matrix is the negative of Player B's.
- There is no pure strategy Nash equilibrium in this game because each player can profitably deviate by choosing the opposite of their opponent's choice.
- However, there is a mixed strategy Nash equilibrium where both players randomize their choices with equal probability.
- Both players randomly choose heads with probability ½and tails with probability ½ .
- In this equilibrium, each player's expected payoff is zero, and neither player has an incentive to deviate from their strategy.

These examples demonstrate different strategic situations with infinite strategy spaces, where players make decisions along a continuum or with mixed strategies. They illustrate how equilibrium outcomes can arise from strategic interactions even in seemingly simple game.

CHAPTER 4

$2 \times n$ and $m \times 2$ Games

4.1 $2 \times n$ Games:

A game in which two players alternately make moves is known as a 2*n game. The game terminates when there are no more moves available. The number of movements allotted to each player is indicated by the notation "2 x n". A game of Tic-Tac-Toe, for instance, is regarded as a 2 x 5 game since each player receives five movements.

Eg: In a 2-player, 2 x n game, each player has two strategies, and the outcome depends on the combination of strategies chosen by both players.

Imagine that Player 1 and Player 2 are playing a game where they each have two strategies: Up and Down.

		Player 2	
		Up	**Down**
Player 1	**Up**	**5, 5**	**2, 4**
	Down	**4, 2**	**3, 3**

The payoffs are as follows:

- If both players choose Up, Player 1 receives a payoff of 5 and Player 2 receives a payoff of 5.
- If Player 1 chooses Up and Player 2 chooses Down, Player 1 receives a payoff of 2 and Player 2 receives a payoff of 4.
- If Player 1 chooses Down and Player 2 chooses Up, Player 1 receives a payoff of 4 and Player 2 receives a payoff of 2.

- If both players choose Down, Player 1 receives a payoff of 3 and Player 2 receives a payoff of 3.

These payoffs are represented in the format (Player 1's payoff, Player 2's payoff) for each combination of strategies chosen by the two players.

4.2 $m \times 2$ Games:

A m x 2 game is a game in which m people participate, and each player gets to make two movements before the turn is passed to the next player. The number of players and the total number of movements made by each player are indicated by the notation "m x 2". For example, in a four-person game of poker, each player has two moves before the next player takes the table, making it a 4 x 2 game.

Eg: Player 1 and Player 2 are the two players in the game in this case. Player 2 has two strategies, Fight and Negotiate, whereas Player 1 has four: Attack, Defend, Retreat, and Surrender. There are two payoffs for any combination of strategies that the players choose; Player 1's payoff is represented by the first number, while Player 2's payoff is represented by the second.

- **Attack vs. Fight:** If Player 1 makes the Attack choice and Player 2 makes the Fight choice, Player 1 may benefit greatly (3) if their aggressive play is successful, but they may also suffer if Player 2 counters well (Player 2 receives 5). This is a reflection of aggression's risk-reward trade-off.
- **Protect vs. Fight**: Depending on how well Player 1 defends against Player 2's attack, both players may receive moderate payoffs (4 each) if they decide to protect and Player 2 decides to fight. This illustrates a balanced exchange in which both players use caution and defensiveness.
- **Retreat vs. Fight:** In this scenario, Player 1 could minimize their losses by carefully retiring (Player 1 receives 2), while Player 2 receives.
- **Fight vs. Surrender:** Should Player 1 decide to fight and Player 2 decide to surrender, Player 1 may lose a lot of money (Player 1 receives 1) as a result of their unwavering surrender, while Player 2 receives a hefty payout (3). This is what happens when someone gives in to aggression.
- **Attack or Negotiate:** In this scenario, if Player 1 decides to Attack and Player 2 choose to Negotiate, Player 1 may receive retaliation (Player 1 gets 1) if Player 2 replies tactfully, while Player 2 receives a moderate payout (3) as a result of negotiation.
- **Defend vs. Negotiate:** If Player 1 choose to defend and Player 2 decides to negotiate, both players may be able to work together to negotiate and come to a mutually advantageous compromise in which Player 2 and Player 1 each receive 2.
- **Retreat vs. Negotiate:** If Player 1 decides to retreat and Player 2 decides to negotiate, Player 1 may be able to avoid conflict and engage in negotiation to obtain a positive outcome (Player 1 receives 5), while Player 2 will receive a reduced payout (1).

- **Surrender vs. Negotiate:** In this scenario, if Player 1 choose to Surrender and Player 2 decides to Negotiate, Player 1 may use bargaining to obtain a fair result (Player 1 receives 4), while Player 2 uses their position to obtain a large payout (5).

Table Illustration

Player 2

Player 1		**Fight**	**Negotiate**
	Attack	**3, 5**	**1, 3**
	Defend	**4, 4**	**2, 2**
	Retreat	**2, 2**	**5, 1**
	Surrender	**1, 3**	**4, 5**

In this match:

- If Player 1 select Attack and Player 2 select Fight, Player 1 will receive a payout of 3, while Player 2 will receive a payout of 5.
- Player 1 receives a payout of 1 and Player 2 receives a payout of 3 if Player 1 choose Attack and Player 2 selects Negotiate.
- If Player 1 select Defend and Player 2 select Fight, Player 1 will receive a payout of 4, while Player 2 will also receive a payout of 4.
- Player 1 receives a payout of two and Player 2 receives a payout of two if they decide to defend and negotiate, respectively.
- If Player 1 select Retreat and Player 2 select Fight, Player 1 will receive a payout of 2, while Player 2 will receive a payout of 2.
- In the event where Player 1 selects Retreat and Player 2 choose Negotiate, Player 1 will receive a payout of five, while Player 2 would receive one.

- If Player 1 choose for Surrender and Player 2 select Fight, Player 1 will receive a payout of 1, while Player 2 will receive a payout of 3.
- If Player 1 opt for Surrender and Player 2 select Negotiate, Player 1 will receive a payout of 4, while Player 2 will receive a payout of 5.

The payoffs for every combination of tactics that the two players choose are shown in this table. The payoffs for Player 1 and Player 2 are indicated by the numbers in cell.

CHAPTER 5
APPLICATIONS AND CONCLUSIONS

Game theory has been used to study wide variety of human behaviors. It was initially developed in economics to understand a large collection of economicbehaviors including behaviors of times, markets and consumers.

Game theory plays a major role in the study of oligopolies (industries containing only a few firms) and also bargaining.

Applications in Oligopolies:

1. Pricing Strategies: In oligopolistic markets, companies must carefully choose their pricing to optimize profits while taking competitor' responses into account. Different pricing strategies, like collaboration, price leadership, and price competition, are better understood thanks to game theory. The Bertrand competition model, for instance, examines how businesses compete on price while taking competitors' strategic objectives to undercut them and gain market share into account.

2. Strategic Investments: Companies operating in oligopolies frequently allocate resources to capacity expansion, advertising, and research and development. Game theory models the strategic interactions between organizations and offers insights into investment decisions. The investment game framework, for example, helps clarify the dynamics of successive investment choices and strategic commitments.

3.Collusion and Cartels: For the purpose of antitrust law and market regulation, it is essential to understand the dynamics of collusion and cartels among oligopolistic enterprises. Repetitive games and cartel stability analysis are two examples of game theory models that are used to assess the viability of collusive agreements and the efficacy of regulatory actions.

In the area of bargaining, the fundamental ideas of game theory are also applied. Both bargaining theory and game theory include several subfields. The most well-known of these is evolutionary game theory, which provides a game model in which players select their strategy by making mistakes in the game.

The military uses game theory to help in planning and decision-making by considering the possible outcomes of various actions.

- **Planning Battles:** Imagine organizing your forces or strategizing your attack while planning a battle. What the enemy might do and how to counter it can be predicted with the aid of game theory.

- **Preventing War:** States make decisions about how to intervene to prevent other states from initiating a war by using game theory. It's as if we're asking ourselves, "What will happen if we do this? What then ought to we do?"
- **Selecting weaponry:** The selection of weaponry that a nation should purchase is aided by game theory. Selecting the right weapons to be secure is aided by considering what other nations might do.
- **Combating Terrorism:** Terrorism campaigns are planned using game theory. It assists in determining the most effective responses to assaults and deterring terrorist strikes.

With this study, we were able to gain a thorough understanding of game theory. The phrase "game theory" itself prompted us to learn more about it. There are interdependent decisions everywhere, game theory has broad applicability, and its ideas are straightforward.

Furthermore, game theory broadens our understanding of strategic conduct in increasingly complicated contexts by encompassing games with infinite strategies, continuous strategy spaces, and incomplete information in addition to classic finite games. The study of strategic interactions in both finite and infinite games is based on the principles of equilibrium, which continue to be important to this approach.

All things considered, game theory is an effective instrument for evaluating strategic decision-making, forecasting results, and formulating plans of action in a variety of situations. Researchers, legislators, and practitioners can make better decisions and negotiate strategic interactions in their particular domains by knowing the fundamentals of game theory and its applications.

Game theory has several applications in everyday life, as we discovered via further study of the subject. Through the use of game theory, we may enhance our ability to make decisions and get insight into conflict situations. I think future research in game theory will provide more advancements.

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