FUZZY SET THEORY

AND

ITS APPLICATIONS

Dissertation submitted in partial fulfilment of the requirement for the

MASTER OF SCIENCE IN MATHEMATICS

by

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2021- 2023

DECLARATION

I, Farzeen A, hereby declare that this dissertation entitled "FUZZY SET THEORY AND ITS APPLICATIONS" is an authentic record of the original work done by me under the guidance of Disna Mary Joseph, Assistant Professor, Department of Mathematics , Bharata Mata College, Thrikkakara. I also declare, that this dissertation has not been submitted by me fully or partially for the awards of any degree, diploma, title or recognition earlier.

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M.Sc. Mathematics

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CERTIFICATE

This is to certify that the project entitled **'FUZZY SET THEORY AND ITS APPLICATION '** submitted for the partial fulfilment requirement of Master's Degree in Mathematics is the original work done by Farzeen A during the period of the study in the Department of Mathematics, Bharata Mata College, Thrikkakara under my guidance and has not been included in any other project submitted previously for the award of any degree.

disna mary

supervisor

Place: Thrikkakara

Date:

ACKNOWLEDGEMENT

First and foremost I express my sincere gratitude towards the abiding presence and grace of God Almight.

I sincerely express my gratitude to .... ,..........,Department of Bharata Mata College, Thrikkakarafor guiding and helping me to complete this work.
I also express my profound gratitude to Dr. Lakshmi C, Head, the Department of Mathematics for her wholehearted cooperation and valuable advices.

Finally, I am also thankful to all our teachers of our department and the staff of the library,Bharata Mata College for the encouragement for completing this work.

Farzeen A

Contents

INTRODUCTION

Fuzzy set theory has been shown to be a useful tool to describe situations in which the data are imprecise or vague.Their fuzzy logic offers highly valuable flexibility for thinking because in the real world we frequently run into situations where we are unable to discern whether the state is true or false.This allows us to take into account any situation's inaccuracies and uncertainties.

The majority of our traditional tools for formal reasoning, computing, and modeling have a clear, deterministic, and exact nature. Crisp refers to yes-or-no type as opposed to more- or less-type. An element in set theory can either be a member of a set or not. In other words, each component should have a certain essence. For the majority of circumstances in actual life, however, this accuracy cannot be anticipated because; Real-world circumstances are frequently not rigidly predetermined and cannot be properly explained.A human person could never simultaneously recognize, interpret, and comprehend as much information as would be needed to fully describe an actual system.

One of the many disorganized changes in science and mathematics this century relates to the idea of uncertainty or ambiguity.The conventional wisdom holds that science should seek for certainty in all of its components, including precision, specificity, sharpness, etc. As a result, uncertainty is considered to be unscientific (including imprecision, non-specificity, consistency, etc.). The current perspective holds that uncertainty is necessary for research; it is not only an inescapable pestilence but also has a huge usefulness.

The fundamental mathematical foundation of fuzzy set theory and its most significant applications will be covered in this project. Fuzzy set theory, neural network theory, and evolutionary programming have all come to be referred to as "computational intelligence" or "soft computing" since 1992. These regions naturally have a very close association with one another. However, in this project, fuzzy sets, fuzzy set theory, and its practical applications will take center stage.

Chapter 1

PREREQUISITES

1.1 CRISP SETS (ORDINARY SETS/CLASSICAL SETS)

In daily conversation, we frequently refer to groups of related items, such as a deck of cards, a cricket team, etc. We also encounter collections in mathematics, such as those of prime numbers, lines, and natural numbers.If we look at the collections below:

1. Prime numbers less than 10.
2. The vowels in English alphabets.
3. The solution of the equation $x^2 - 7x + 10 = 0$.

Each of these examples is a well-defined collection of objects. In that way we can decide whether a given particular object belongs to a given particular collection or not. Therefor we shall say that **an ordinary set is a well-defined collection of objects**.

Sets are usually denoted by capital letters A, B,X, Y, Z, etc and the elements or members of set are represented by small letters a, b, x, y, z, etc [3]. If a is an element of a set A, we denote it as $a \in A$, reading 'an element of A' or 'a belongs to A' and if b is not an element in A then we denote it as $b \notin A$ [3].

Two sets A and B are said to be **equal** if they have exactly the same elements and we write $A = B$ [3] . Otherwise they are said to be **unequal** and we write $A \neq B$.

If A and B are any two sets and every element of A is also an element of B, then A is said to be a **subset** of B, denoted by $A \subseteq B$ or equivalently, B is said to be **superset** of A, denoted by $B \supseteq A$ [3] . In other words, $A \subseteq B$ if whenever $a \in A$ then $a \in B$. If A is a subset of B, but A is not equal to B, then A is a **proper subset** of B, denoted by $A \subset B$ or equivalently, B is a **proper superset** of A denoted by $B \supset A$ [3].

In a given situation, we typically have to deal with the basic set's components and sub components that apply to that situation. The system of natural numbers $\mathbb{N}$ and its subsets, such as the set of all prime numbers, the set of all odd numbers, and so on, are of particular importance to us as we explore the mathematical system of numbers. The **universal set** is the name of this fundamental set. In other words, a universal set in set theory is a set that contains all things, including itself. Typically, the universal set U and all of its subsets (A, B, X, Y, etc.) are designated by the letter U.

A classical set is differentiated in two ways:

One method is to list the elements that belongs to the set; often called the **roster or tabular form**, where all elements are listed, the elements being separated by commas and are enclosed within braces {} . For example, set of all natural numbers which divide 12 is enumerated as {1,2,3,4,6,12}.

Another method to describe the set analytic, that is, in the **set builder form**, where all the elements of a set possess a single common property which is not possessed by any element outside the set [3]. For example, D={x : $xisanaturalnumber$; 3¡ x ¡10}

A classical set can also be defined by listing the member elements by using **characteristic function (indicator function)** that can be defined on the set X having the value 1 of A and a 0 for all others of X not in A .

That is, the characteristic function of a subset A of a set X (universal set) is a function I_A**:**$X \longrightarrow \{0,1\}$ defined as,a

$$I_A(\text{x})=\begin{cases} 1 & if x \in A \\ 0 & if x \notin A \end{cases}$$

1.1.1 OPERATIONS ON CRISP SETS

For two sets A and B and universal set X,

1. **Union:** A∪B= {x ∈X: x ∈A$or x$ ∈B}[4]
2. **Intersection:**A ∩B= {x ∈X: x ∈A$and x$ ∈B}[4]
3. **Compliment:** A'={x∉X: x ∉A} [4]

4. **Difference:** $A-B=\{x \in X: x \in A, x \notin B\}$ [4]

1.1.2 FUNDAMENTAL PROPERTIES OF CRISP SETS

For two sets A and B and universal set X,

- **Commutative property:**

  $A \cup B = B \cup A$ [4]

  $A \cap B = B \cap A$ [4]

- **Associative property:**

  $(A \cup B) \cup C = A \cup (B \cup C)$ [4]

  $(A \cap B) \cap C = A \cap (B \cap C)$ [4]

- **Distributive property:**

  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ [4]

  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ [4]

- **Idempotent law:**

  $A \cup A = A$ [4]

  $A \cap A = A$ [4]

- **Absorption law:**

  $A \cup (A \cap B) = A$ [4]

  $A \cap (A \cup B) = A$ [4]

- **Identity law**($\emptyset$ - identity element)

  $A \cup \emptyset$ = A[4]

  A$\cap\emptyset$ = $\emptyset$ [4]

- **De Morgan's law :**

  $(A \cap B)' = A' \cup B'$ [4]

  $(A \cup B)' = A' \cap B'$ [4]

Chapter 2

FUZZY SETS

2.1 BASIC DEFINITIONS

(The word "Fuzzy" means vagueness or ambiguity)

Let X be a universal set. If X is a collection of objects denoted commonly by x, then a fuzzy set A in X is a set of ordered pairs:

A={(x,$\mu_A(x)$):x∈X} [4]

Where $\mu_A(x)$ [or $A(x)$] is called the **membership function or grade of membership** (also degree of compatibility or degree of truth) of x in A that maps X to [0,1]. [4]

(if A is a classical subset of X then $\mu_A(x)$ is 1 when x ∈A$and 0 when$x∉A$and \mu_A(x)$ is identical to the characteristic function of a non-fuzzy set).

Elements with a zero degree of membership are normally not listed.

According to the crisp set theory, there are two groups of people in each given realm of discourse: members (those who unquestionably belong to the group) and non-members (those who unquestionably do not). It was insufficient in the sense that it failed to assign the degree to which each individual belongs to a given set under consideration. It was in such a context that L A Zadeh introduced fuzzy set theory, a generalized form of classical set theory.

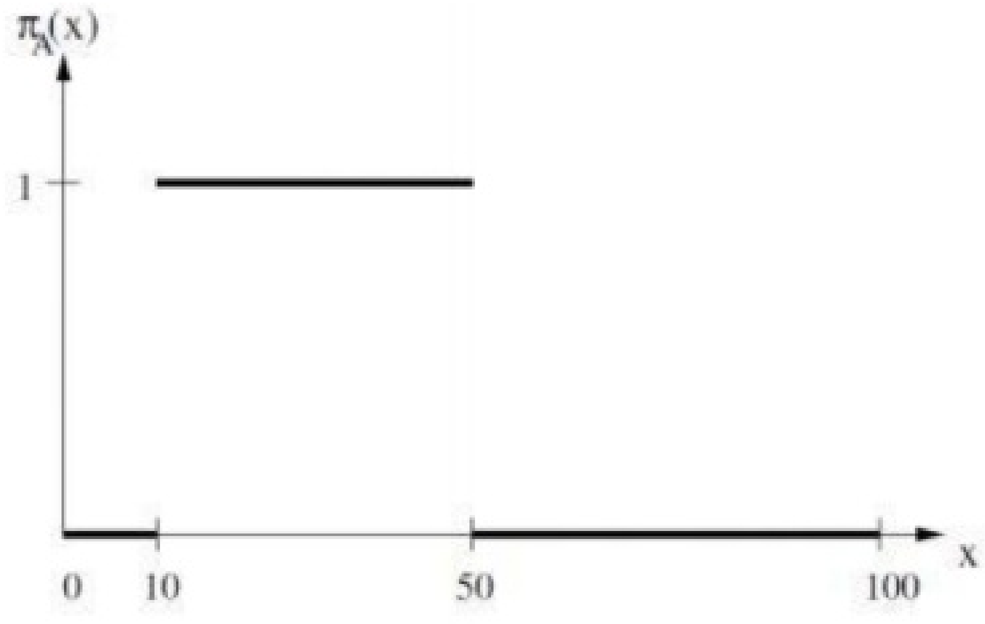
Fuzzy sets are used to provide a more reasonable interpretation of linguistic variables.

A fuzzy set is a generalization of the classic set theory that uses the membership function to yield a number between 0 and 1, which denotes the degree to which an item x has membership in the set A.

2.1.1 MEMBERSHIP FUNCTION

- The membership function serves as the focal point of fuzzy sets.
- Each element in the domain has a degree of membership that is connected to the associated fuzzy set using the function.
- A membership function serves as another way to describe two-valued sets.
- Consider the domain X of floating-point values in the range [0,100], for instance.

Defining the crisp set $A \subset X$ of the floating- numbers in the range [10,50] [3] The figure given below is the illustration of Membership function for Two-Valued



Sets.

All membership functions must satisfy the following constraints:

1. It must have a 0 and 1 boundary on the bottom and top, respectively.

2. A membership function must have a [0,1] range.

3. $\mu_A(x)$ must be unique for each x∈X.

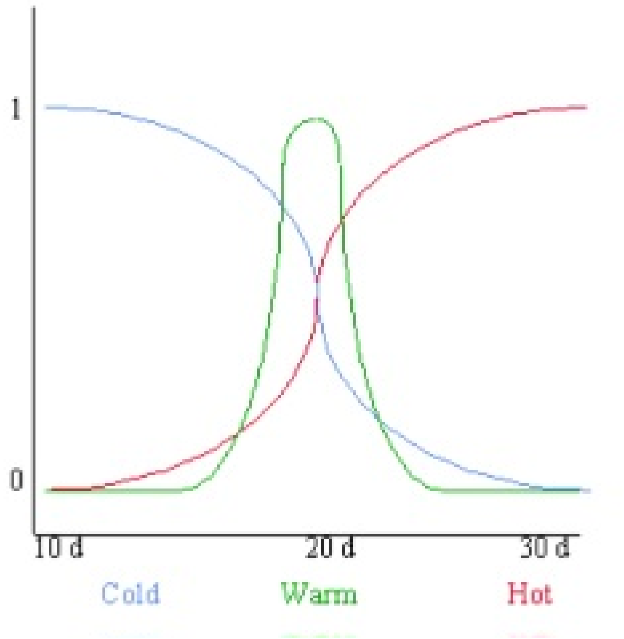
   In other words, a single element cannot translate to several degrees of membership for a single fuzzy set.

2.1.2 EXAMPLES OF FUZZY SETS

1. Let X={a, b, c, d, e}

   Let A be the fuzzy set of "smart" students, where "smart" is fuzzy term. A={(a,0.4),(b,0.5),(c,1),(d,0.9),(e,0.8)} Here A indicates that the smartness of a is 0.4 ans so on.

2. The graph demonstrates how one may use a 20 degree threshold as a starting point for assigning fuzzy values to various temperatures.



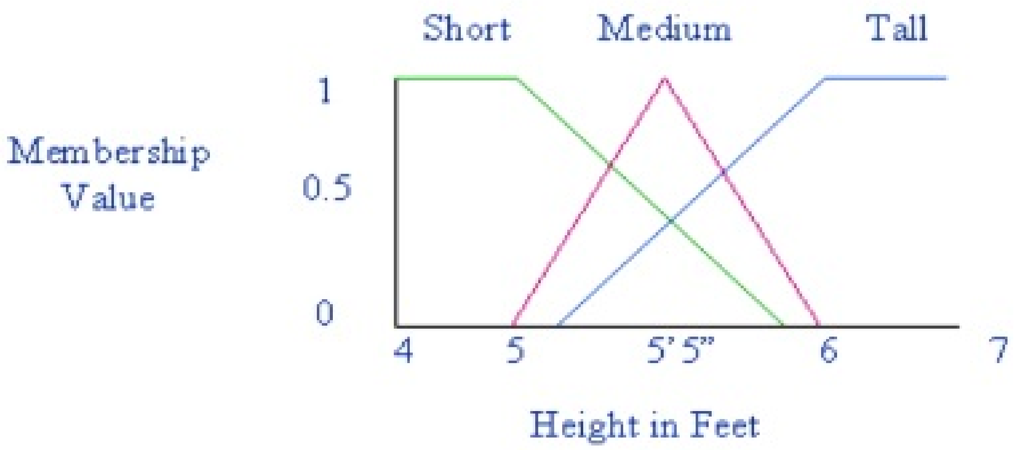
it is given by;

10 degrees = {1 c,0 w,0 h}

20 degrees = {0.5 c,1 w,0.5 h}

30 degrees = {0.15 c,0.15 w,0.85 h}

3. Take a look at the figure below. A person who is 5'5" tall is considered to be a member of the "medium" person category with a membership value of 1, as well as the "short" and "tall" categories with a value of 0.25.



4. Let X= ℝ, the set of all real numbers be the universal set under consideration.

   Then, A={(x,$\mu_($$x$)($x$)):$x$∈X}

   $i.e.$,A= {real number near 0}; $where$,

   $\mu_A(x)= \frac{1}{1+x}$ is a fuzzy set on X.

2.1.3 CRISP SETS AND FUZZY SETS

- Crisp sets are defined by crisp boundaries.
- **Fuzzy sets are defined by indeterminate boundaries.**
- Crisp sets contain the precise location of the set boundaries.
- **In fuzzy sets, there exists an uncertainty about the set boundaries.**
- Crisp sets has exactly one membership function.

- **Fuzzy sets can have an infinite number of membership functions to represent it.**
- An element in a crisp set is either a member of the set or it is not.
- Items in **Fuzzy** sets allow items to only be partially included in the set.
- Crisp sets describe values as either 1 or 0, as they are of the YES or NO type
- **Fuzzy sets are MORE or LESS type and thus defines values between 0 and 1.**

2.1.4 OPERATIONS ON FUZZY SETS

Given X to be the universe of discourse and A and B to be fuzzy sets with $\mu_A(x)$ and $\mu_B(x)$ are their respective membership function, the fuzzy set operations are as follows:

- **Standard Fuzzy Union** of A and B, is defined by

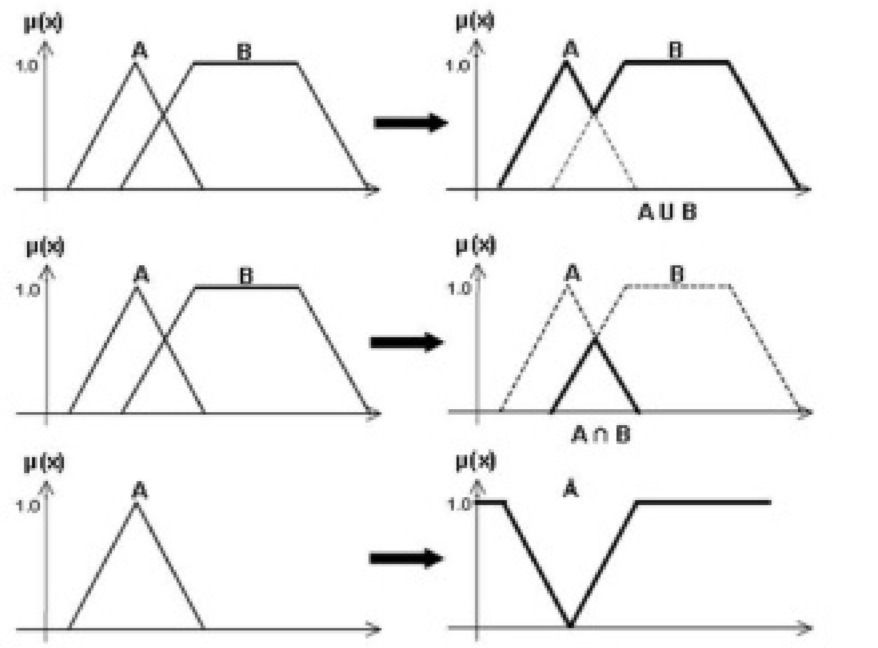
  $\mu_{A\cup B}(x)$ = max [$\mu_A(x)$, $\mu_B(x)$] [4]

- **Standard Fuzzy Intersection of** A and B, denoted as $A \cup B$, is defined by

  $\mu_{A\cap B}(x)$ = min [$\mu_A(x)$, $\mu_B(x)$] [4]

- **Standard Fuzzy Complement (negation)** of A, denoted as A, is defined by $\mu_{\bar{A}}$ (x) = 1- $\mu_A(x)$ [4]

The figure given below is the illustration of standard fuzzy operations:Union, Intersection and compliment



Example: Let $X = \{x_1, x_2, x_3\}$, $A = \{(x_1, 0), (x_2, 0.5), (x_3, 0.7)\}$ and $B = \{(x_1, 0.3), (x_2, 0.5), (x_3, 0.6)\}$

Then,

- $A \cup B = \{(x_1, 0.3), (x_2, 0.5), (x_3, 0.7)\}$
- $A \cap B = \{ (x_1, 0), (x_2, 0.5), (x_3, 0.6)\}$
- $\bar{A} = \{(x_1, 1), (x_2, 0.5), (x_3, 0.3)\}$

- Fuzzy sets follow the same properties of crisp sets except for the Law of Contradiction and the Law of Excluded Middle.

  That is,

  $A \cap \bar{A} \neq \varphi$ [Law of Contradiction]

  $A \cup \bar{A} \neq X$[Law of Excluded Middle]

2.2 T-CONORM AND T-NORM

2.2.1 T-CONORM (fuzzy union)

The general fuzzy union of two fuzzy sets A and B is given by a function

U: [0,1] $\times$ [0,1]$\longrightarrow [0,1]$[3]

$thatsatisfiesatleastthefollowingaxiomsforalla,b,c\in [0,1]are$;

- $Axiom$-1: $U(a, 0) = a$ (boundary condition) [3]
- $Axiom$-2: if $a<$a'andb<b'$then$U(a,b) $\leq$U(a',b')$(monotonicity)$[3]
- $Axiom$-3:$U(a, b)$=$U(b, a)$ (commutativity) [3]
- $Axiom$-4: $U(U(a, b),c) = U\ (a, U(b,c))(associativity)$[3]

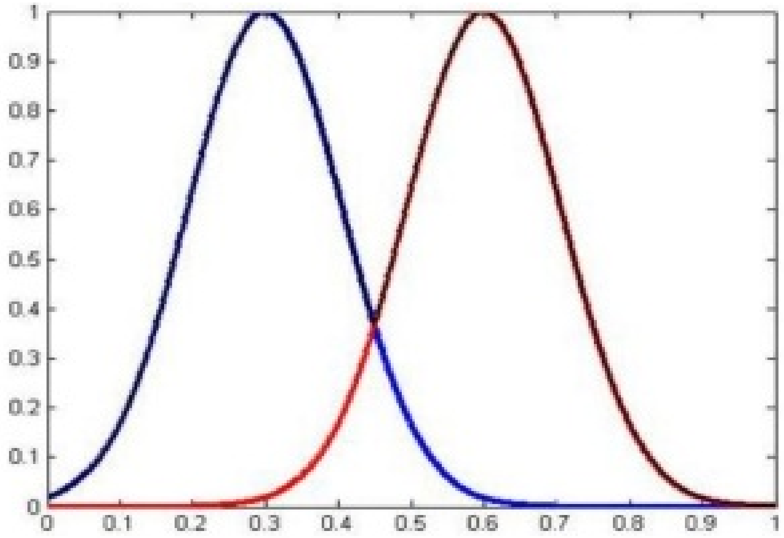
These axioms form the Axiomatic Skeleton for fuzzy union.

The other requirements for fuzzy unions are expressed by the following axioms:

- $Axiom - 5$: U is a continuous function. [3]
- $Axiom - 6$:$U(a, a)$ =a (idempotency) [3]

Examples: t-conorms that are frequently used as fuzzy union (each defined for all a, $b \in [0,1])are$;

1. Standard Union: $U(a, b)$= max (a, b) [3]



Standard fuzzy union

Function U satisfies *Axiom* 5 and *Axiom* 6 along with the axiomatic skeleton.

2. Algebraic Sum: $U(a, b) = a + b$ - ab

   Function U satisfy *Axiom* 5, i.e, it is continuous but not idempotent (*Axiom* 6)

3. Bounded Sum: $U(a, b) = \min\ (1, a + b)$ [3]

   Function U is continuous but it is not idempotent (*Axiom* 6)

4. Drastic Union: $U(a, b) = \begin{cases} a; & when \quad b = 0 \\ b; & when \quad a = 0 \\ 1; & otherwise \end{cases}$ [3]

otherwise U is a continuous function (*Axiom* 5) but it's not idempotent (*Axiom*6)

2.2.2 T-NORM (fuzzy intersection)

The fuzzy intersection t-norm is defined by a function, I: [0,1] $\times$ [0,1]$\longrightarrow [0,1]$that satisfies at least the following axioms for all a, b, $c \in [0,1] are$; [3]

- *Axiom*-1: $I(a, 0) = 0$(boundary condition) [3]
- *Axiom*-2: if a ¡ a' and b ¡b' then $I(a, b)$ $\leq$I(a',b')(*monotonicity*)[3]
- *Axiom*-3:$I(a, b)$=$I(b, a)$ (commutativity) [3]
- *Axiom*-4: $I(I(a, b)$,$c)$ = I (a, $I(b$,c)) (associativity) [3]

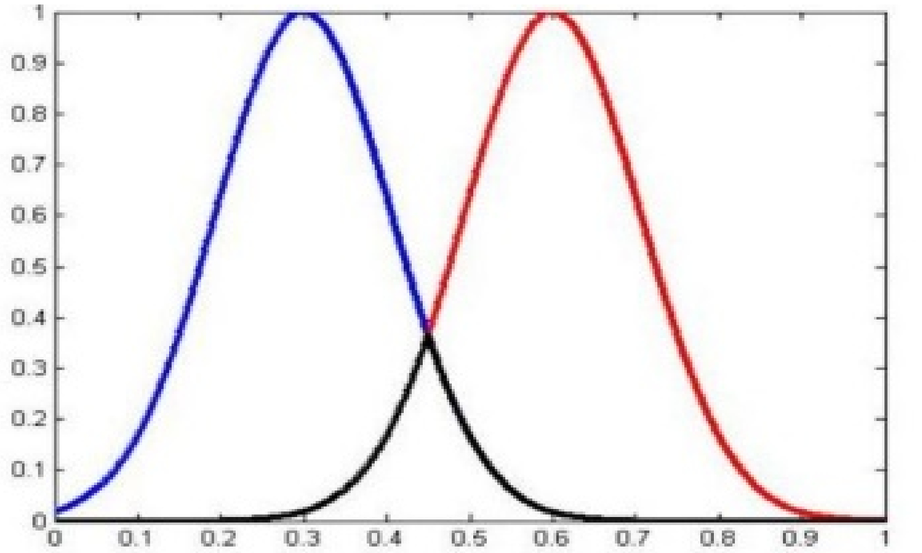
These axioms form the Axiomatic frame for fuzzy intersection.

The other requirements for fuzzy intersection are expressed by the following axioms:

- $Axiom - 5$: $I$is a continuous function. [3]
- $Axiom - 6$:$I(a, a)$ =a (idempotency) [3]

Examples: t-conorms that are frequently used as fuzzy intersection (each defined for all a, $b \in [0,1]) are$;

1. Standard intersection: $I(a, b)$= mix (a, b)



Standard fuzzy intersection

Function I satisfies all the six axiomatic requirements.

2. Algebraic product: $I(a, b) = ab$

   Function I satisfy *Axiom* 5, i.e it is continuous but not idempotent (*Axiom* 6)

3. Bounded difference: $I(a, b) = \max\,(0, a + b\text{-}1)$

   Function I is continuous (*Axiom* 5) but it is not idempotent (*Axiom* 6)

4. Drastic intersection: $I(a, b) = \begin{cases} a; & when \quad b = 1 \\ b; & when \quad a = 1 \\ 0; & otherwise \end{cases}$ [3]

otherwise I is a continuous function (*Axiom* 5) but it is not idempotent (*Axiom*6)

2.3 FUZZY COMPLEMENTS

The fuzzy complement is defined as a function C:[0,1]$\longrightarrow$ $[0, 1]$[3],which satisfies the following axioms.To produce meaningful fuzzy complements,function C must satisfy at least the following two axiomatic requirements,

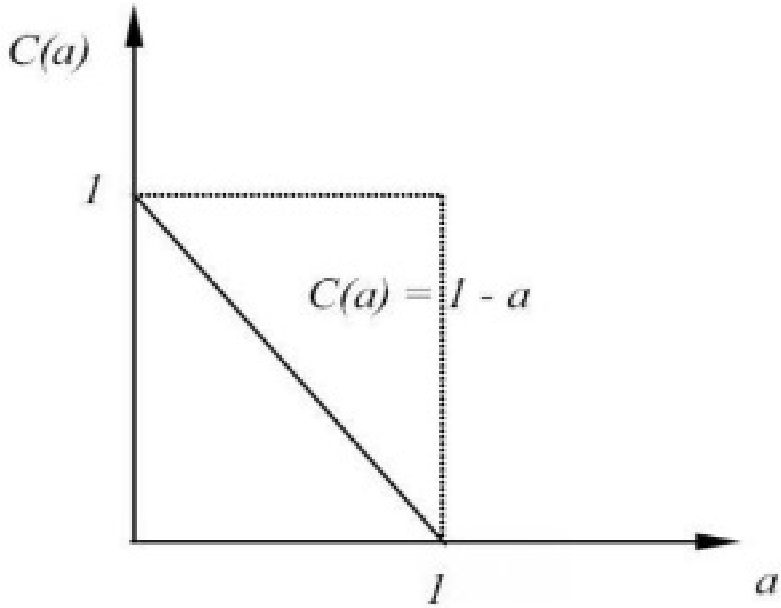
- *Axiom* 1:$C(0) = 1$ and $C(1) = 0$ (boundary condition) [3]
- *Axiom* 2: if a¡b, and a,b $\in [0, 1]$thenC(a) $\geq$C(b)(*monotonicity*)[3]

The violation of either of these axioms would result in addition of some functions totally unacceptable as fuzzy complements. Hence *Axiom* 1 and *Axiom* 2 are known as the *Axiomatic frames* for fuzzy complements.

Certain additional desirable requirements listed as axioms of fuzzy complements is given by,

- *Axiom* 3:C is continuous function. [3]
- *Axiom* 4: C is involute. i.e.,$C(C$(a)) $= a$,∀a∈ $[0, 1]$[3]

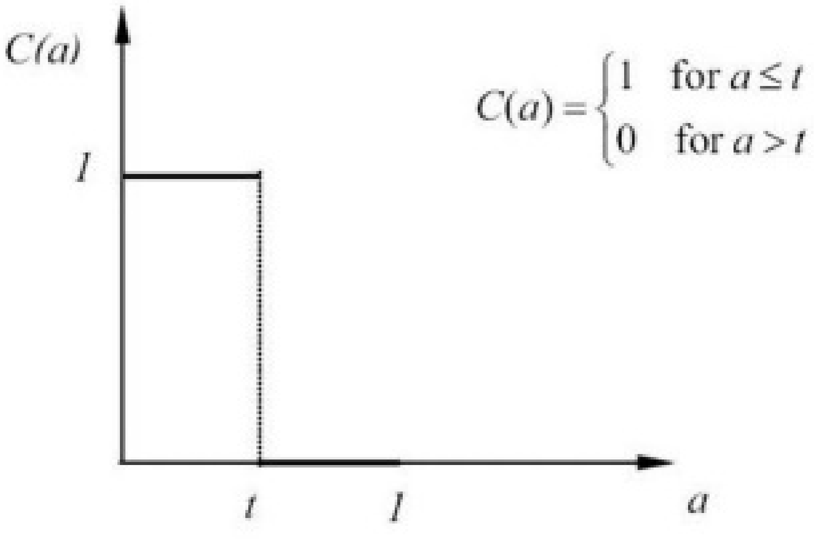
EXAMPLE 1: Standard function $\mu_{\bar{A}}\ (x)$ = 1- $\mu_A(x)$ satisfies all the four



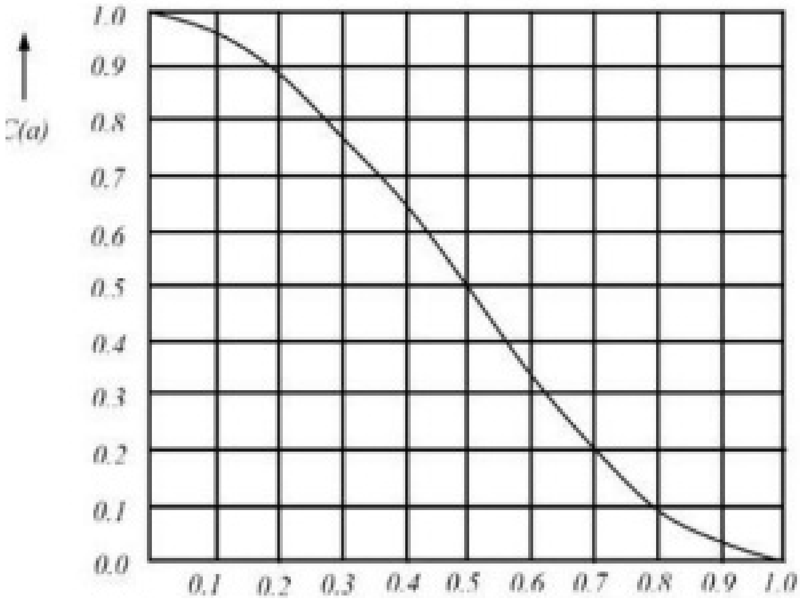
axioms.

Above figure represents the standard fuzzy compliment

EXAMPLE 2: The function C(a) $=\begin{cases} 1 & for \quad a \le t \\ 0 & for \quad a > t \end{cases}$ satisfies the axioms only.



EXAMPLE 3: The function C(a) =0.5$(1 + cos\pi$a) satisfies axiomatic skeleton



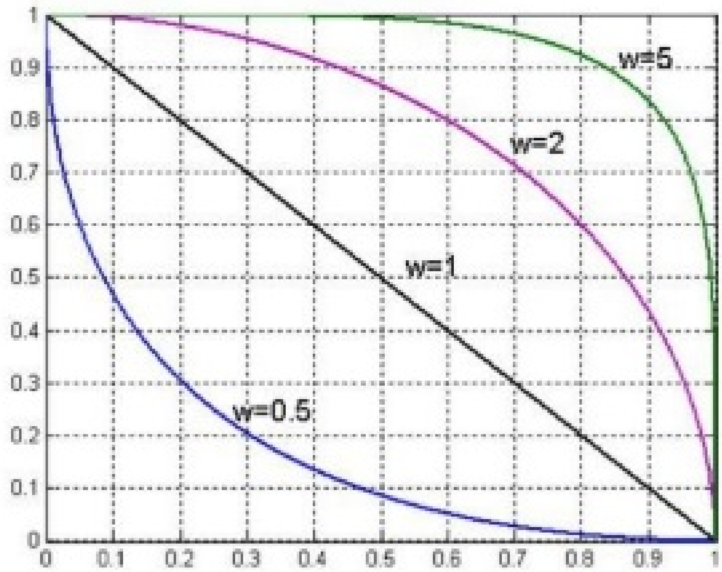
and

axiom 3. Since C(0,75) = 0.5, the value of $C(0.33)$ for a = 0.33 is not equal to 0.33.

Therefore axiom 4 does not follow now.

EXAMPLE 4: Yager's function $C_w(\mathrm{a}) = (1 - a^w)^{1/w}$

where w$\in (-1, \infty)$satisfies all the axioms.



2.4 FUZZY SET - CHARACTERISTICS

The main characteristic of membership function include **normality, height, support, core, cut, unimodality and cardinality.**

NORMALITY OF A: A fuzzy set A is normal if that set has an element with a membership function of 1. That is, ∃x∈ A;$\mu_A(x) = 1$ [2]

Height of A: The height of a fuzzy set A is defined as the supremum of the membership value of membership function [2]. That is;

$Height(A) 0r$H(A)= $sup\{\mu_A(x)$; x∈X}[2]

Support of A: The support of fuzzy set A is the set of all elements in the universal of discourse, X, that belongs to A with non-zero membership value [2]. It is denoted by support$A$ or $S(A)$, it is given by,

$S(A)$= {x∈X:$\mu_A(x)$ >0 }

Example: Let X ={ x_1, x_2, x_3, x_4 } and $A = \{(x_1, 0.2), (x_2, 0.5), (x_3, 0), (x_4, 1)\}$

Here, $S(A) = \{x_1, X_2, x_4\}$.

Core of A: Let A be a fuzzy set on X. The core of A is the set of all elements in the domain that belongs to A with membership degree 1 [2].

$Core(A) = \{x$∈X:$\mu_A(x) = 1$ }

Example: Let X ={ x_1, x_2, x_3, x_4 } and $A = \{(x_1, 0.2), (x_2, 0.5), (x_3, 0), (x_4, 1)\}$

Here, $Core(A) = \{x_4\}$ [2].

Note: Fuzzy singletons are fuzzy sets with $mu_A(x) = 1$ as their support, also known as fuzzy single points.

α **cut strong** α **Cut:** Let A be a fuzzy set on X and $\alpha \in [0, 1]$.[3]

Then α cut of A denoted as αA, is defined by

αA= {x∈X:$\mu_A(x) \geq \alpha$ }

The strong α cut of A, denoted as $\alpha + A$, is defined by

$\alpha + A$ = {x∈X:$\mu_A(x) > \alpha$ } [3]

Example: Let X= { x_1, x_2, x_3 } and $A = \{(x_1, 0), (x_2, 0.3), (x_3, 0.7)\}$ some α cuts are,

0_A ={x∈X:$\mu_A(x) \geq 0$} = {x$_1, x_2, x_3$ }

0.2_A ={x∈X:$\mu_A(x) \geq 0.2$} = {x$_1, x_3$ }

Some strong α cuts are,

$0 + A$ = {x∈X:$\mu_A(x) >0$ } = { x_2, x_3 }

$0.3 + A$ ={x∈X:$\mu_A(x) >0.3$ } ={ x_3 }.

Unimodality: If a fuzzy set's membership function only has one possible value, it is said to be unimodal. In other words, the function has a single maximum.

Cardinality: The cardinality of fuzzy set A, for a finite domain X is defined as,

$Card(A) = \sum_{x \in X} \mu_A(x)$ [3]

Normalization: A fuzzy set is normalized by dividing the membership function by the height of the fuzzy set. That is;

$Normalized(A) = \frac{\mu_A(x)}{Height(x)}$

Level set

Let A be a fuzzy set on X.

The level set of A denoted by $\Delta(A)$ is defined as the set of all levels $\alpha \in [0,1]$,which represents the distinctα cuts of A.

$\Delta(A)$= $\{\alpha \in [0,1]$:$\mu_A(x) = \alpha$, for some x∈X}[3]

Example: Let X ={ x_1, x_2, x_3 } and $A = \{(x_1, 0.1), (x_2, 0.9), (x_3, 0.7)\}$

Then $\Delta(A)$ = { 0.1,0.7,0.9}.

Chapter 3

FUZZY LOGIC

An approach to thinking that mirrors human reasoning is fuzzy logic. Fuzzy logic uses an approach that mimics how humans make decisions, which entails considering all possible outcomes between the digital values YES and NO. It is a general term for the idea of incomplete truth.

Based on the whim of relative graded membership and drawing inspiration from human perception and comprehension, fuzzy logic is a theory. In 1965, Lotfi A. Zadeh released his seminal study on fuzzy sets. Information derived through computational perception and understanding, which is ambiguous, imprecise, vague, partially true, or lacking distinct limits, can be dealt with using fuzzy logic. Fuzzy logic enables the incorporation of hazy human judgments into computational issues. Additionally, it offers a useful method for resolving issues with various criteria and better option evaluation. New computing methods based on fuzzy logic can be used in the development of intelligent systems for decision making, identification, pattern recognition, optimization, and control[3] .

features that differentiate fuzzy logic and traditional logical system is given by:

1. A suggestion in a two-valued logical system is either true or false. A suggestion

in a multi-valued logical system can be either true or false, or it might have a middle ground truth value that could be a component of either a limited or infinite truth value set T. The truth values in fuzzy logic are permitted to span the fuzzy subsets of the set T. A truth value in fuzzy logic, such as "very true," for instance, may be construed as a fuzzy subset of the unit interval if T is the unit interval. In this way, a fuzzy truth value could be thought of as a vague description of a numerical truth value.

2. The indication of a predicate in two-valued logic is required to be a non-fuzzy subset of the discourse universe, and this constraint forces predicates to be crisp. In fuzzy logic, the predicates can be either sharp—for example, "mortal," "even," and "father of"—or more widely fuzzy, such as "ill," "tired," "large," "tall," "much heavier," and "friend of."

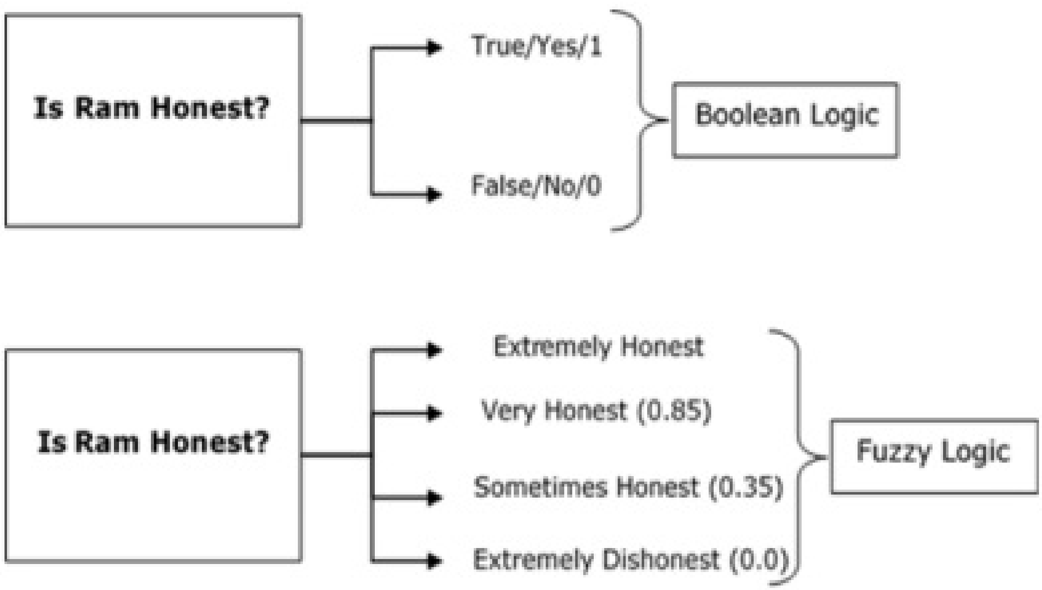
3. Only two quantifiers—"all" and "some"—are permitted in logics with two and multiple values. The use of fuzzy quantifiers, such as "most," "many," "several," "few," "a lot of," "often," "occasionally," and so on, is also permitted by fuzzy logic. The cardinality of one or more fuzzy or non-fuzzy sets can be loosely described by such quantifiers, which can be thought of as fuzzy numbers. A fuzzy quantifier can be thought of from this angle as a second-order fuzzy predicate. Based on this view, fuzzy quantifiers may be used to represent the meaning of propositions containing fuzzy probabilities and thereby make it possible to manipulate probabilities within fuzzy logic [3].

4. Using "not," "very," "more or less," "extremely," "slightly," "much," "a little,"

and other similar expressions, fuzzy logic offers a way to convey the meaning of both non-fuzzy and fuzzy predicate-modifiers. Consequently, a system for computing with linguistic variables—i.e., variables whose values are words or sentences in a natural or synthetic language—is created. For instance, the linguistic variable "Age" is used when its values are considered to be "young," "old," "very young," "not very old," and so on.

5. In two-valued logical systems, a statement can be qualified by adding a truth value—"true" or "false"—a modal operator, like "possible" or "necessary," and an intentional operator, like "know" or "believe."

EXAMPLE: Figure given below shows the difference between Boolean logic and Fuzzy logic.



Chapter 4

APPLICATIONS OF FUZZY SET THEORY

Fuzzy set theory has applications in many fields, including robotics, artificial intelligence, computer science, control engineering, decision theory, expert systems, logic, management science, and operations research.

Fuzzy logic has a wide range of uses, including facial pattern recognition, air conditioners, washing machines, vacuum cleaners, transmission systems, control of subway systems, unmanned helicopters, knowledge-based power system optimization systems, weather forecasting systems, models for new product pricing or project risk assessment, medical diagnosis and treatment plans, and stock trading. In a variety of industries, including control systems engineering, image processing, power engineering, industrial automation, robotics, consumer electronics, and optimization, fuzzy logic has been applied with success. Long-dormant scientific domains have recently regained their vibrancy thanks to this area of mathematics. The kinds of applications of fuzzy logic have undergone a major transformation over the last two decades.Non engineering applications have grown in number, visibility and importance. Among such applications are applications in medicine, social sciences, policy sciences, fraud detection systems, assessment of credit-worthiness systems and economics [2].

In this chapter we intend to give description on application of fuzzy logic in medical diagnosis.

4.1 MEDICAL DIAGNOSIS

One industry where the use of fuzzy set theory was identified relatively early, in the middle of the 1970s, was medicine. The ambiguity present in the process of disease diagnosis has repeatedly been the subject of applications of fuzzy set theory in this discipline. We look at some fundamental problems with these applications in this section. New medical technology have expanded the amount of information available to doctors, making it more challenging to categorize various sets of symptoms under a single name and choose the best course of treatment. Even the same disease might manifest itself remarkably differently in various persons and at various disease stages. Additionally, one symptom may be indicative of multiple diseases, and the coexistence of multiple diseases in one patient may alter the typical symptom pattern for any one of them. The most accurate and helpful explanations of illness entities frequently make use of linguistic expressions that are utterly ambiguous. One source of ambiguity and uncertainty in the diagnostic process is the medical knowledge of the association between symptoms and diseases, while another is the medical knowledge of the patient's condition. The patient's prior history, physical examination, lab test results, and other investigative techniques like X-rays and ultrasonics are often how the doctor learns about the patient. The degree of accuracy in the information offered by each of these sources varies. The patient may provide a subjective, inflated, understated, or incomplete previous history. During the physical examination, mistakes could be made and symptoms could go unnoticed. Laboratory data are frequently imprecise, and the precise line dividing normal from diseased conditions is frequently ambiguous. The results of X-rays and other related procedures must be correctly interpreted. As a result, the doctor can only determine the patient's condition and symptoms with a certain amount of accuracy.Despite the ambiguity around the patient's reported symptoms and the lack of clarity surrounding the symptoms' relationship to a specific condition, it is imperative that the doctor choose the diagnostic classification that suggests the best course of treatment. Textbf fuzzy sets have been used to attempt to mimic this challenging and crucial medical diagnosis process in the hopes of better understanding and teaching it. The extent to which each model tries to address various complicating factors in medical diagnosis, such as the significance of symptoms, the various symptom patterns associated with various disease stages, relationships between diseases themselves, and the stages of hypothesis formation, preliminary diagnosis, and final diagnosis within the diagnostic process, varies. These models also serve as the foundation for computerized medical expert systems, which are typically created to help doctors diagnose a certain class of disorders. Numerous methods for modeling the diagnostic process have made use of the fuzzy set architecture. The doctor's medical expertise is shown as a hazy association between symptoms and diseases in Sanchez's [1979] approach. Thus, given the fuzzy set A of the symptoms observed in the patient and the fuzzy relation R representing the medical knowledge that relates the symptoms in set S to the diseases in set D, then the fuzzy set B of the possible diseases of the patient can be inferred by means of the compositional rule of inference[1].

$$B = (A \circ R)[1] \tag{4.1}$$

$B(d) = maxs \in S[min(A(s), R(s, d)]$

for each $d \in D$. The membership grades of observed symptoms in fuzzy set A may represent the degree of possibility of the presence of the symptom or its rigorousness. The degrees of plausibility with which we can associate each pertinent diagnostic label with the patient are indicated by the membership grades in the fuzzy set B. The biggest relationship should be formed between the fuzzy relation Q on the set of patients and symptoms and the fuzzy relation R of medical knowledge. Relation Q on the set P of patients and S of symptoms and the fuzzy relation T on the sets P of patients and D of diseases, then

$$T = (Q \circ R)[1] \tag{4.2}$$

Thus, relations Q and T may represent, respectively, the symptoms that were present and diagnoses consequently made for a number of known cases.The relationship between symptoms and diseases that was established in the earlier diagnosis can be specified using the gathered medical expertise by solving the fuzzy relation equation (4.2) for R. In order to avoid finding a connection that is more precise than our information ensures, the maximal solution to (4.2) must be chosen for R. This may result in instances when R indicates a stronger correlation between symptoms and disease than is actually the case. It could be essential to interpret the outcomes of applying relation R to a particular set of symptoms as a diagnostic hypothesis rather than as a finalized diagnosis.Applications of fuzzy set theory in medicine are by no means restricted to medical diagnosis. Other applications involve, for example, fuzzy controllers for various medical devices, fuzzy pattern recognition and image processing for analysis of X-ray images and other visual data, and fuzzy decision making for determining appropriate therapies [1].

CONCLUSION

In this project I studied about Fuzzy sets which have great utility in simplified representation of uncertainty and its applications in various fields. It has been used in numerous branches of mathematics, including topology, analysis, clustering, control theory, graph theory, measure theory, and algebra.

This project helped me to learn foundations of Fuzzy sets which had been helpful in representing vague concepts expressed in natural language. The wide applicability of this concept is impressive enough to have a future endeavour on this project.

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