CHAOS

UNDERGRADUATION PROJECT

STUDY OF CHAOS IN NON LINEAR SYSTEM AND IT'S ILLUSTRATION BY COMPUTER SIMULATION

PREPARED BY

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BSC PHYSICS(VOC: COMPUTER APPLICATION)

BHARATA MATA COLLEGE THRIKKAKARA

RE- ACCREDITED NY NAAC WITH A+ GRADE

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A PROJECT REPORT ON

STUDY OF CHAOS IN NONLINEAR SYSTEM AND ITS ILLUSTRATION BY COMPUTER SIMULATION

DEPARTMENT OF PHYSICS

SUBMITTED BY

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CERTIFICATE

This is to certify that the project report entitled “Study of chaos in nonlinear system and it's illustration by computer simulation” is an authentic work carried out by Nitha Hanan, Reg.No.200021039183, for the partial implementation of the requirement For the award of a degree BACHELOR OF SCIENCE IN PHYSICS through the Post Graduate Department of Physics, Bharata Mata College, Thrikkakara, affiliated to Mahatma Gandhi University, Kottayam, Kerala.

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DECLARATION

I Nitha Hanan, hereby declare that this project report entitled “Study of chaos in nonlinear system and it's illustration by computer simulation” is an authentic work carried out during my course of study under the guidance of Dr. Lini Devassy, Assistant Professor, Post Graduate Department of Physics, Bharata Mata College, Thrikkakara.

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Content

INTRODUCTION

The study of chaos in nonlinear systems began in the 1960s with the work of mathematicians and physicists like Edward Lorenz, Benoit Mandelbrot, and Mitchell Feigenbaum. They were interested in understanding the behavior of systems that exhibit extreme sensitivity to initial conditions, where small differences in initial conditions can lead to drastically different outcomes.

Lorenz, in particular, is credited with discovering the butterfly effect, which describes how a small change in one part of a system can have large and unpredictable effects elsewhere in the system. His work on chaos theory began with his study of weather patterns, where he found that small changes in the initial conditions of a weather system could lead to completely different outcomes, making long-term weather forecasting impossible.

Mandelbrot, on the other hand, developed the concept of fractals, which are self-similar patterns that repeat at different scales. Fractals are found in many natural systems, from coastlines and mountain ranges to the branching patterns of trees and blood vessels. They have since been used in a wide range of fields, from computer graphics and animation to finance and medicine.

Feigenbaum's work focused on the behavior of nonlinear systems as they approach a state of chaos, and he discovered a universal constant that describes how quickly the system approaches chaos. This constant, now known as the Feigenbaum constant, has since been found to appear in a wide range of nonlinear systems.

A non-linear system is a mathematical or physical system in which the relationships between the system's variables are not proportional. In other words, when the system is subject to a change, the response of the system is not directly proportional to the input.

Non-linear systems are more complex than linear systems, and their behavior can be difficult to predict. The equations that govern non-linear systems often involve higher powers or products of the variables, making them much more difficult to solve than linear systems. Non-linear systems can exhibit behaviors such as chaos, bifurcation, and multiple solutions, which are not observed in linear systems. Examples of non-linear systems include weather patterns, biological systems, financial markets, and chaotic systems like the double pendulum. Non-linear systems are often studied using computer simulations and mathematical models.

An example of a non-linear system of equations is the Lorenz system, which describes a simplified model of atmospheric convection:

$dx/dt = \sigma(y - x)$

$dy/dt = x(\rho - z) - y$

$dz/dt = xy - \beta z$

where x, y, and z are the state variables, and σ, ρ, and β are parameters that determine the behavior of the system.

This system exhibits chaotic behavior, meaning that small changes in the initial conditions can lead to very different outcomes. It is often used as a model for complex systems in physics, biology, and engineering.

Another example of a non-linear system is the Lotka-Volterra equations, which describe the interaction between predator and prey populations in an ecosystem:

$$dx/dt = \alpha x - \beta xy$$

$$dy/dt = \delta xy - \gamma y$$

where x is the population of the prey species, y is the population of the predator species, and α, β, δ, and γ are parameters that determine the behavior of the system.

This system exhibits oscillatory behavior, meaning that the populations of the two species fluctuate over time in a cyclic pattern. It is often used as a model for predator-prey dynamics in ecology.

The study of nonlinear systems began with the discovery of the Lorenz attractor, a mathematical model of atmospheric convection, by meteorologist Edward Lorenz in the 1960s. The Lorenz attractor demonstrated that even small changes in initial conditions can lead to drastically different weather patterns, and it introduced the concept of chaos theory

In nonlinear systems, chaos refers to a phenomenon in which small differences in initial conditions can lead to vastly different outcomes over time. This means that even small changes in the starting state of a system can result in dramatically different behavior as time progresses.

Chaos arises due to the sensitive dependence on initial conditions, which is also known as the butterfly effect. This means that even tiny variations in the initial state of a system can lead to significant changes in its future behavior. In chaotic systems, the dynamics are often unpredictable, and small perturbations can cause large deviations from the expected behavior.

Examples of chaotic systems include weather patterns, fluid flow, and the behavior of some mechanical systems. Despite the apparent randomness of chaotic systems, they can exhibit underlying patterns and structures that can be studied and understood through mathematical techniques such as chaos theory and nonlinear dynamics.

The study of chaos in nonlinear systems is a field of mathematics and physics that explores the behavior of complex systems that exhibit chaotic behavior. Chaos theory involves the study of dynamic systems that are highly sensitive to initial conditions, and can exhibit seemingly random behavior. This field has applications in a wide range of fields, including weather forecasting, economics, and engineering.

One way to investigate chaos in nonlinear systems is through computer simulations using mathematical models. These models can help researchers understand the underlying mechanisms that drive chaotic behavior and identify patterns and structures within chaotic systems.

Another approach to studying chaos in nonlinear systems is through the design and analysis of nonlinear electronic circuits. These circuits are designed to exhibit chaotic behavior, and can provide insights into the behaviour of other non linear systems.

Chaos theory is a branch of mathematics and science that studies the behavior of systems that are highly sensitive to initial conditions. In other words, small changes in the starting conditions of a system can lead to large and unpredictable outcomes. This sensitivity to initial conditions is also known as the "butterfly effect," in which a butterfly flapping its wings in one part of the world can potentially cause a tornado in another part of the world.

The butterfly effect and chaos theory are two related concepts in the field of mathematics and physics. The butterfly effect is a phenomenon that describes how small changes in initial conditions can lead to vastly different outcomes in complex systems, such as weather patterns, financial markets, or even the behavior of living organisms. The term "butterfly effect" was coined by meteorologist Edward Lorenz, who observed that a tiny change in the initial conditions of a weather simulation caused a drastically different outcome.

Chaos theory is concerned with understanding how complex and seemingly random behavior can arise in systems that are fundamentally deterministic, meaning that their behavior can be predicted with perfect accuracy if their initial conditions are known precisely. Examples of systems that exhibit chaotic behavior include weather patterns, the motion of planets, and the behavior of fluids.One of the key concepts in chaos theory is the idea of a chaotic attractor, which is a set of points in phase space (the space of all possible states of a system) that the system tends to approach over time. The attractor can have a fractal structure, meaning that it has self-similar patterns at different scales.

Chaotic systems are characterized by their sensitivity to initial conditions, their strange attractors, and their unpredictability. Strange attractors are mathematical structures that define the long-term behavior of chaotic systems. They are called "strange" because they have fractal dimensions, which means they have an infinitely complex and self-repeating structure.

Chaos in non-linear systems is a phenomenon where small changes in initial conditions can lead to very different outcomes in the long run. This sensitivity to initial conditions is known as the butterfly effect, where a small change in the initial state of a system can cause large differences in the outcome of the system over time.

Non-linear systems are those in which the output is not directly proportional to the input, but rather is determined by complex interactions between the various components of the system. These systems can exhibit chaotic behavior, even though their equations of motion are deterministic and predictable.

Chaotic systems can arise in many different contexts, ranging from the physical world to abstract mathematical models. In fact, chaos is a ubiquitous feature of complex systems, and it can be found in everything from the weather patterns to the behavior of financial markets.

One common example of a chaotic system is the double pendulum. The double pendulum consists of two pendulums attached to each other, and its behavior is highly unpredictable and chaotic. Even small changes in the initial conditions of the system can lead to drastically different outcomes over time.

Another example of a chaotic system is the Lorenz attractor, which was first studied by meteorologist Edward Lorenz in the 1960s. The Lorenz attractor is a set of three ordinary differential equations that model the behavior of a simplified atmospheric system. Despite its simplicity, the Lorenz attractor exhibits complex, chaotic behavior that is highly sensitive to initial conditions.

Chaos theory has important applications in fields such as physics, engineering, economics, and biology, and has led to the development of new mathematical techniques and tools for analyzing and predicting the behavior of complex systems.

CHAPTER-2

SIGNIFICANCE OF CHAOS THEORY

Chaos theory has had a significant impact on our understanding of physical systems and has led to new insights into the behavior of complex systems in nature. Some of the key contributions of chaos theory to physics include:

Understanding the behavior of chaotic systems: Chaos theory has provided a framework for understanding the complex behavior of systems that exhibit chaos. By analyzing the structure of strange attractors and identifying the underlying mechanisms that give rise to chaotic behavior, physicists have gained new insights into the nature of complex systems.

Predicting the behavior of physical systems: While chaotic systems are inherently unpredictable in the long term, chaos theory has provided tools for predicting the behavior of physical systems over short time scales. For example, weather forecasting relies on chaotic models of atmospheric behavior to make short-term predictions about future weather patterns.

Exploring the limits of predictability: Chaos theory has also shed light on the fundamental limits of predictability in physical systems. The butterfly effect, which describes the sensitivity of chaotic systems to small changes in initial conditions, has highlighted the fact that even the most accurate measurements of initial conditions can only provide limited predictability over time.

Developing new mathematical tools: The study of chaotic systems has also led to the development of new mathematical tools and techniques for analyzing complex systems in physics. These include techniques such as fractal analysis and chaos theory-based methods for analyzing time series data.

Overall, chaos theory has had a profound impact on our understanding of physical systems, and it has provided new insights into the behavior of complex systems in nature. By revealing the underlying mechanisms that give rise to chaotic behavior, chaos theory has opened up new avenues for research and exploration in physics.

APPLICATION OF CHAOS THEORY IN PHYSICS

Fluid dynamics: The flow of fluids is a non-linear system, and chaos theory helps to understand the chaotic behavior of turbulence in fluids.

Here are some specific examples of the use of chaos theory in fluid dynamics:

Turbulence: Turbulence is a chaotic phenomenon that occurs in fluid systems when the flow becomes highly irregular and unpredictable. Chaos theory can help us understand the dynamics of turbulent flows, predict their behavior, and develop strategies to control turbulence.

Mixing: The mixing of fluids is a crucial process in many industrial applications, such as chemical processing and fuel combustion. Chaos theory can help us understand the complex dynamics of mixing processes, optimize mixing strategies, and develop new technologies for efficient mixing.

Vortex dynamics: Vortices are spinning regions of fluid that can have complex, chaotic behavior. Chaos theory can help us understand the dynamics of vortex systems, predict their behavior, and design more efficient vortex-based systems.

Fluid-structure interactions: Fluid-structure interactions occur when a fluid interacts with a solid object, such as a wing or a turbine blade. The complex dynamics of these interactions can be understood using chaos theory, which can help us design more efficient and reliable fluid-structure systems.

Biological fluid dynamics: Fluid dynamics plays a crucial role in many biological systems, such as blood flow in the circulatory system and fluid flow in the respiratory system. Chaos theory can help us understand the complex dynamics of biological fluid systems, predict their behavior, and develop new technologies for diagnosing and treating diseases.

Quantum mechanics: Quantum mechanics is a non-linear system, and chaos theory helps to understand the behavior of quantum systems under different conditions.

Here are some specific examples of the use of chaos theory in quantum mechanics:

Quantum chaos: Quantum chaos is a field of research that studies the behavior of quantum systems that exhibit chaotic behavior. Chaos theory provides a framework for understanding the complex dynamics of these systems and predicting their behavior.

Quantum computing: Quantum computers rely on the principles of quantum mechanics to perform calculations. Chaos theory can help us understand the behavior of complex quantum systems and design more efficient quantum algorithms.

Quantum cryptography: Quantum cryptography relies on the principles of quantum mechanics to provide secure communication. Chaos theory can help us understand the behavior of quantum systems used in quantum cryptography and design more secure cryptographic protocols.

Quantum chaos in condensed matter physics: Condensed matter physics is the study of the behavior of matter in its condensed phases, such as solids and liquids. Chaos theory has been used to study the complex dynamics of condensed matter systems and understand the behavior of quantum systems in condensed matter.

Quantum chaos in nuclear physics: Nuclear physics is the study of the behavior of atomic nuclei. Chaos theory has been used to study the complex dynamics of nuclear systems and understand the behavior of quantum systems in nuclear physics.

Celestial mechanics: Celestial mechanics involves the study of the motion of celestial bodies in space. It is a non-linear system, and chaos theory helps to understand the chaotic behavior of celestial objects like asteroids, comets, and planets.

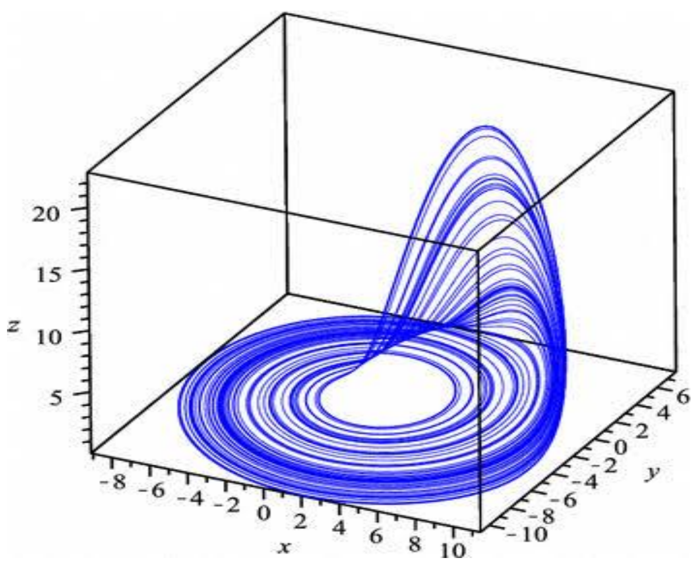
Non-linear optics: Non-linear optics is the study of the interaction of light with matter under high-intensity conditions.In nonlinear optics, the response of a material to light is not linearly proportional to the intensity of the light. This can lead to phenomena such as frequency mixing, harmonic generation, and parametric amplification. Chaos theory helps to understand the chaotic behavior of light in non-linear optical systems, which has applications in telecommunications, data storage, and sensing.

Chapter3

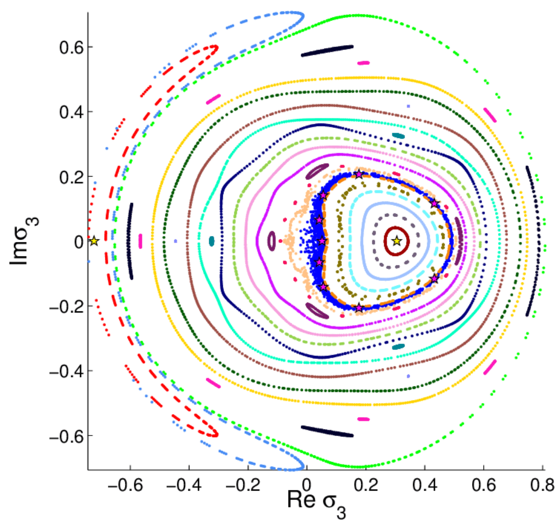
AN ATTEMPT AT ILLUSTRATING CHAOS

There are various methods to illustrate chaos in non-linear systems. Here are a few:

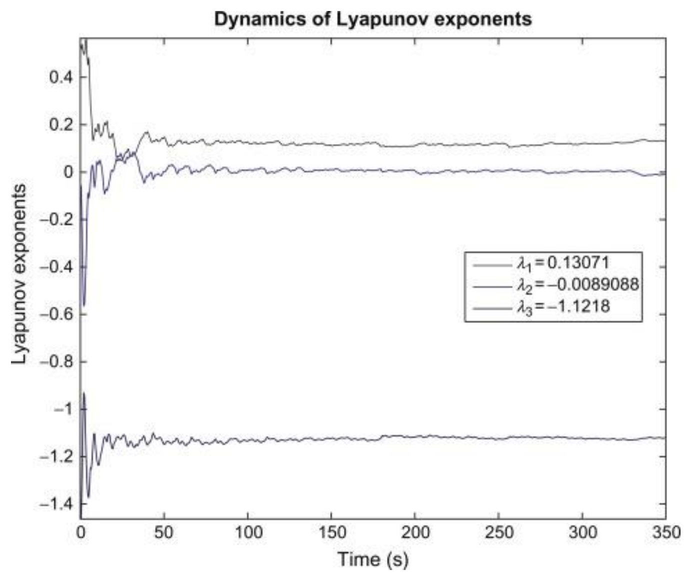
Phase Space Plots: A phase space plot is a graph of the system's state variables (such as position and velocity) against each other. In a chaotic system, the plot will appear to be a random scatter of points.



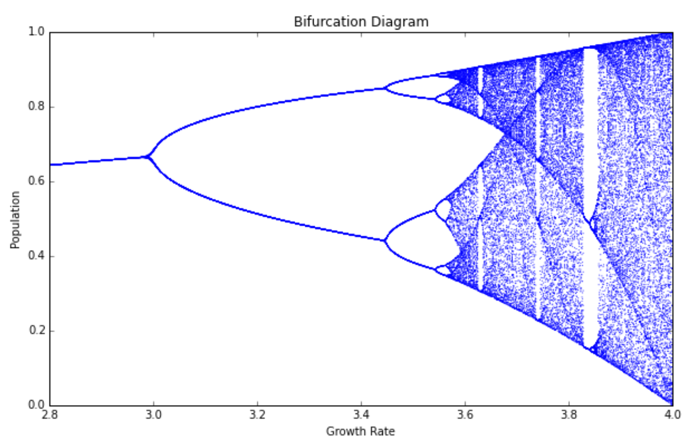
Poincaré maps: A Poincaré map is a projection of a chaotic trajectory onto a lower-dimensional space. By choosing a specific cross-section of the trajectory and plotting the values of the system variables at each intersection, a Poincaré map can be created. The resulting plot often shows a complex, fractal-like pattern.



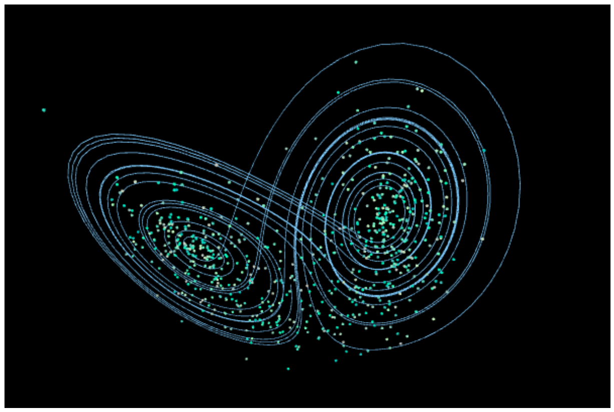
Lyapunov Exponents: Lyapunov exponents are a measure of the rate at which nearby trajectories in phase space diverge. In a chaotic system, the Lyapunov exponents will be positive, indicating that nearby trajectories quickly become separated.



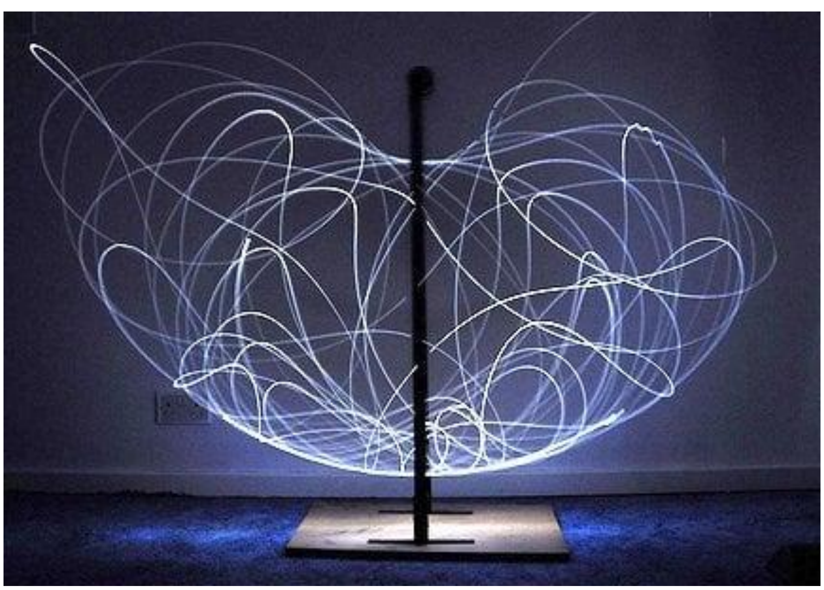
Bifurcation Diagrams: Bifurcation diagrams show how the behavior of a system changes as a control parameter (such as the system's input) is varied. In a chaotic system, the diagram will show a complex pattern of branches and sub-branches.



Strange attractors: A strange attractor is a geometric object that describes the long-term behavior of a chaotic system. In a chaotic system, the attractor will have a fractal-like structure and exhibit sensitive dependence on initial conditions. The Lorenz attractor is a well-known example of a strange attractor.



The Double Pendulum: The double pendulum is a simple mechanical system consisting of two pendulums connected by a rigid rod. While the motion of a single pendulum is predictable and follows a periodic pattern, the double pendulum exhibits chaotic behavior. Even small changes in the initial conditions, such as the starting angle or velocity of the pendulums, can lead to vastly different trajectories over time.



Chapter 4

DYNAMIC RESPONSE OF A NONLINEAR SYSTEM AS REPRESENTED BY PYTHON

Python is a high-level, interpreted programming language that is widely used in various fields, including web development, scientific computing, data analysis, artificial intelligence, and more. Python was first released in 1991 by Guido van Rossum, and it has since grown in popularity due to its ease of use, simplicity, and powerful features.

Python is designed to be human-readable, which means that its syntax is easy to read and understand. This makes it an excellent choice for beginners who are just starting to learn how to program. Python code is also highly readable because of its use of whitespace, which makes it easy to see the structure of the code at a glance.

One of the key features of Python is its large and active community. This community contributes to the development of various libraries and frameworks that extend the capabilities of Python. This means that Python can be used for a wide range of applications, from web development with Flask or Django, to scientific computing with NumPy or SciPy, and machine learning with TensorFlow or PyTorch.

PYTHON FUNCTION PARAMETERS

In Python, parameters are used to pass arguments to a function or method. There are two types of parameters in Python:

Positional parameters: These parameters are passed to a function based on their position or order. The order in which the arguments are passed is important and should match the order of the parameters in the function definition.

Example:

```
def add_numbers(a, b):
    return a + b


result = add_numbers(2, 3) # Here, 2 and 3 are positional parameters

print(result) # Output: 5
```

Keyword parameters: These parameters are passed to a function based on their name. In this case, the order of the arguments does not matter, but the name of the parameter should match the name used in the function definition.

Example:

```
def multiply_numbers(x, y):
    return x * y
```

```
result = multiply_numbers(x=2, y=3) # Here, x and y are keyword parameters
print(result) # Output: 6
```

Python also supports default parameters, which are used to provide a default value to a parameter if no value is passed for that parameter.

Example:

```
def greet(name="John"):
   print("Hello, " + name)
greet() # Output: Hello, John
greet("Jane") # Output: Hello, Jane
```

In this example, the name parameter has a default value of "John". If no value is passed for the name parameter, it defaults to "John".

CHARACTERISTICS OF PYTHON

Python is a popular high-level programming language that is known for its simplicity, readability, and ease of use. Here are some of its characteristics:

Easy to Learn: Python has a simple and easy-to-understand syntax. It is a high-level language and its code is much more readable than other programming languages.

Interpreted: Python is an interpreted language which means that the code is executed line by line. There is no need for compilation before running the code.

Object-Oriented: Python supports object-oriented programming, which means that it allows you to create objects that have attributes and methods.

Dynamically Typed: Python is dynamically typed, which means that you do not need to declare the data type of a variable before assigning a value to it.

Platform Independent: Python is a platform-independent language, which means that you can run your code on any platform that has a Python interpreter installed.

Large Standard Library: Python has a large standard library that provides various modules and functions for performing different tasks, such as working with files, networking, and web development.

Third-Party Libraries: Python has a vast ecosystem of third-party libraries that can be easily installed using package managers like pip. These libraries can help you perform complex tasks with ease.

Flexibility: Python is a versatile language that can be used for a wide range of applications, such as web development, data analysis, machine learning, and automation.

Open-Source: Python is an open-source language, which means that its source code is freely available and can be modified and distributed by anyone.

Python can be used to simulate the behavior of the Lorenz system and visualize its chaotic behavior. For example, using Matplotlib, one can create 3D plots of the system's attractor, which is a geometric representation of the system's long-term behavior.

Another popular tool for studying chaotic systems in Python is the fractal geometry library, Fractals. Fractals can be used to generate and visualize complex, self-similar patterns that are often observed in chaotic systems.

Python provides a rich ecosystem of libraries for scientific computing, such as NumPy, SciPy, Matplotlib, and Pandas, which offer tools for numerical computation, optimization, data visualization, and data manipulation. These libraries enable researchers and students to simulate and explore complex nonlinear systems and study their chaotic behavior.

A chaos equation is a mathematical model that describes the behavior of a point in n-dimensional space using a system of n arbitrary equations. In two-dimensional space, the system consists of two equations describing the coordinates of a point in terms of its x and y positions. The starting values of the x and y coordinates are arbitrary, and the system includes constants that affect the point's movement.

To simulate the behavior of the system, the equations are repeatedly applied to calculate new x and y coordinates n times. Each iteration results in a new set of coordinates, and the system's behavior can be observed over time.

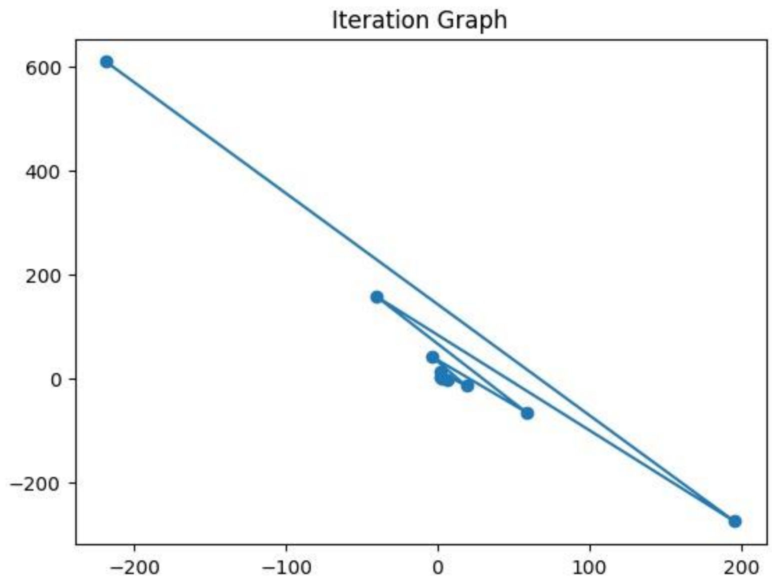
For example, a chaos equation in two-dimensional space might start with an initial point at position [1, 1], and include a constant T = 3. The formula for calculating new x coordinates might be "new_x = x + yT", and the formula for calculating new y coordinates might be "new_y = x - yT". By iterating the equations a number of times, the behavior of the system can be studied and analyzed.

Python provides a rich ecosystem of libraries for scientific computing, such as NumPy, SciPy, Matplotlib, and Pandas, which offer tools for numerical computation, optimization, data visualization, and data manipulation. These libraries enable researchers and students to simulate and explore complex nonlinear systems and study their chaotic behavior.

Python code:

```
import matplotlib.pyplot as plt

n = 10 # number of iterations

x, y = (1, 1) # initial coordinates
```

```
T = 1.5 # arbitrary constant
x_cord = [] #list to store x cordinates
y_cord = [] #list to store y cordinates

for i in range(1, n+1): # repeat n times
    new_x = x + y*T # calculate new x
    new_y = x - y*T # calculate new y
    x, y = (new_x, new_y) # set x, y to new_x, new_y respectively
    x_cord.append(x)    #append the x cordinate to the list
    y_cord.append(y) #append the y coordinate to the list
    print(f"{i}. iteration [x, y] = [{x}, {y}]")

#constructing graph:
plt.title("Iteration Graph")
plt.scatter(x_cord, y_cord) #creating a scatter plot from cordinates
plt.plot(x_cord, y_cord) #connecting the scattered points using line
plt.show() #to display the plot
```



one popular nonlinear system that exhibits chaotic behavior is the Lorenz system, which is a set of three differential equations that describe the evolution of a system of three variables over time. The Lorenz system is named after the mathematician Edward Lorenz, who discovered its chaotic behavior in the 1960s.

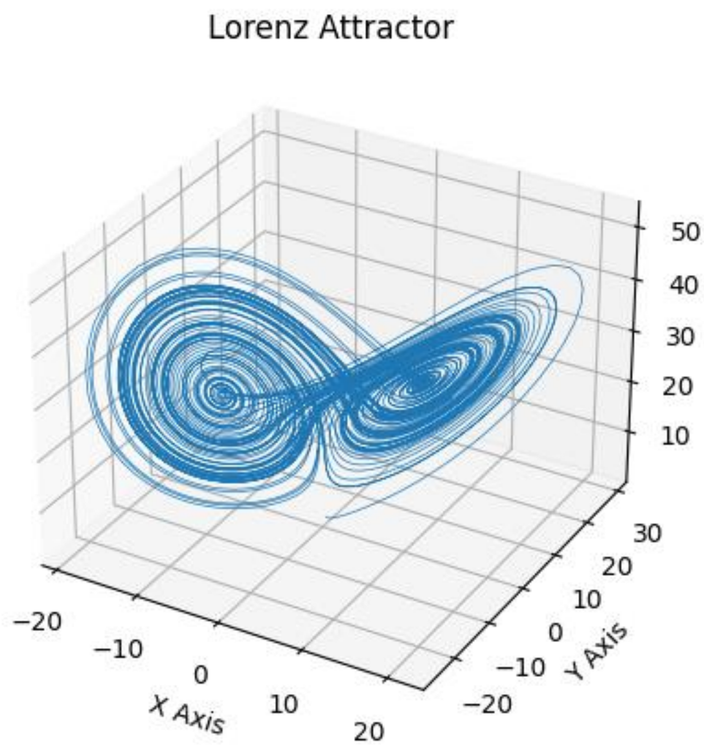
Using Python, one can simulate the Lorenz system and visualize its chaotic behavior using Matplotlib

Python code:

```
import matplotlib.pyplot as plt

import numpy as np


def lorenz(xyz, *, s=10, r=28, b=2.667):
    """
    Parameters
```

```
    ----------
    xyz : array-like, shape (3,)
       Point of interest in three-dimensional space.
    s, r, b : float
       Parameters defining the Lorenz attractor.

    Returns
    -------
    xyz_dot : array, shape (3,)
       Values of the Lorenz attractor's partial derivatives at *xyz*.
    """
    x, y, z = xyz
    x_dot = s*(y - x)
    y_dot = r*x - y - x*z
    z_dot = x*y - b*z
    return np.array([x_dot, y_dot, z_dot])


dt = 0.01
num_steps = 10000

xyzs = np.empty((num_steps + 1, 3))  # Need one more for the initial values
```

```
xyzs[0] = (0., 1., 1.05)  # Set initial values

# Step through "time", calculating the partial derivatives at the current point
# and using them to estimate the next point
for i in range(num_steps):
    xyzs[i + 1] = xyzs[i] + lorenz(xyzs[i]) * dt


# Plot
ax = plt.figure().add_subplot(projection='3d')


ax.plot(*xyzs.T, lw=0.5)
ax.set_xlabel("X Axis")
ax.set_ylabel("Y Axis")
ax.set_zlabel("Z Axis")
ax.set_title("Lorenz Attractor")

plt.show()
```


Lorenz Attractor
50
40
30
20
10
30
20
10
0
−10
−20
Y Axis
−20
−10
0
10
20
X Axis

CONCLUSION

The theory of chaos in nonlinear systems has transformed our understanding of the behaviour of complex systems. Through the study of chaos, we have come to appreciate the intricate relationships between seemingly unrelated phenomena and the profound impact of small changes in initial conditions. Chaos theory has expanded our understanding of the natural world and has led to significant advancements in various fields, including physics, engineering, biology, economics, and social sciences. The insights provided by chaos theory have enabled us to make better predictions, develop new technologies, and gain a deeper appreciation of the beauty and complexity of the universe. While much remains to be explored and understood in the field of chaos theory, its impact on our understanding of the world around us is undeniable, and its applications are likely to continue to transform the way we approach problems and make decisions.

Python's ease of use and accessibility make it an ideal tool for students and researchers looking to explore the field of chaos theory. With its extensive documentation, vast online community, and open-source nature, Python provides a valuable resource for anyone interested in studying nonlinear systems and their behaviour. By combining the power of Python with our understanding of nonlinear dynamics and chaos theory, we can unlock new insights into the natural world and revolutionize our approach to solving complex problems.

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