TOPOLOGY IN SET

Dissertation submitted in partial fulfillment of the requirements for the

BACHELOR'S DEGREE IN MATHEMATICS

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DECLARATION

We hereby explicitly state that the project report titled *"TOPOLOGY IN SET"* is a legitimate record of the work we carried out under Dr. Lakshmi C's direction and that it hadn't previously been utilized as a basis for either the conferral of any academic degree, social or other title of similar kind from another institution or board.

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CERTIFICATE

This is to certify that the dissertation titled "TOPOLOGY IN SET" submitted jointly by Josna Anna K Antony, Ashna Ansar, Nandana Rajeev, Meghana K S, Athul Antony Sabu and Sreeram Krishna K S, is a true record of the work carried out by them under my direct guidance at the Department of Mathematics, Bharata Mata College, Thrikkakara, during the period between 2020–2023. This dissertation has never before been submitted elsewhere for another degree.

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ABSTRACT

Chapter 1 : Introduction To Topology
Crucial definitions about the topology in set are presented in this chapter.

Chapter 2: Topological Space
In this chapter we discuss about topological space and related theorems.

Chapter 3 : Subspace And Topology
In this chapter we discuss theorems related to continuity and compact set.

INTRODUCTION

Topology is the study of the characteristics and connections that hold true during continuous transformations of an item, such as stretching, bending, and twisting, but not for gluing or tearing. It is concentrated on the study of spaces and the qualities that withstand ongoing change. Topology has applications in many areas of mathematics, physics, and engineering, such as computer science, robotics, algebraic geometry, analysis, and differential equations. Topological concepts like continuity, compactness, connectivity, and convergence are crucial. The Poincaré conjecture and the classification of surfaces are just two examples of the important theories and findings that topology has enabled.

Topologies can be used to analyze a variety of spatial characteristics, including function continuity, connectivity, and compactness. They are also utilized in industries like physics, computer science and topology data analysis.

Type of topology

- Bus topology
- Star topology
- Mesh topology
- Ring topology
- Hybrid topology

Application topology describes how the parts of an application are connected to one another to form a network or system, and a set is a collection of unique items. Application topology in the context of sets may refer to the relationships between sets.

Neighborhood is referred to as NBD, and it is a key idea in topology. It speaks of a portion of a topological space that has an open set that contains a certain point.

CHAPTER 1

INTRODUCTION TO TOPOLOGY

DEFINITION OF TOPOLOGY

Suppose that we have a set S that is not empty and that W is the collection of all subsets of S that satisfy the three criteria listed below:

a)$\Phi \in W$, $S \in W$.
b) If we consider the intersection of two subsets W_1 and W_2, each of which is an element of W, then their intersection is in W (i.e., if $W_1 \in W$, and $W_2 \in W$, then $W_1 \cap W_2 \in W$).
c) The union of any sub collection of W's elements is in W.

For instance,
If S = {1, 2, 3} and W = 1, 2, and 1, 2, then W is a topology on S since it meets the three requirements, a) W,S W.
b) Additionally, any sub collection of W intersects in W.
c) The union of any subcollection of W's elements is in W.

For instance:
If S = {1, 2, 3} and W ={Φ ,S, {1} , {2} ,{1,2} }, then W is a topology on S since it meets the three requirements,
a) $\Phi \in W$,$S \in W$.
b) Additionally, any sub collection of W intersects in W.
c) The union of any subcollection of W's elements is in W.

Consider another instance,

If $S=\{x,y,z\}$ and $W=\{ \Phi,S ,\{x\} ,\{y\} \}$, then S is not a topology on W because,

a) $\Phi \in W$, $S \in W$

b) In the event where $W_1 \in W$, and $W_2 \in W$, then $W_1 \cap W_2 \in W$.

c) Assume that $W_1 = \{x\} \in W$ and $W_2 = \{y\} \in W$, but $W_1 \cup W_2 \notin W$.

W is not a topology on S as a result.

<u>TOPOLOGICAL SPACE: WHAT IS IT?</u>

Assume that (S,W) is recognised as a topological space if W is a topology on a nonempty set S.

<u>TYPES OF TOPOLOGY</u>

There are 5 distinct kinds of topology:

i) Indiscrete Topology
ii) Discrete Topology
iii) Co-finite Topology
iv) Co-countable Topology
v) Finer and Coarser Topology

(i)The Indiscrete Topology-

If S is a non-empty set, then the collection $I =\{\Phi ,S\}$, which includes the empty set and the entire space, is always a topology for S and is known as the indiscrete topology, and (S,I) is referred to as Indiscrete topological space.

(ii)The Discrete Topology-

If D is the totality of all subsets of S, then D is a topology for S, referred to as discrete space.

Let's use an illustration to clarify:
In the event that $S = \{x, y\}$, then $W = \{\Phi, S, \{x\}, \{y\}\}$ is a discrete topology for S.

(iii) The Co-finite Topology-
Assuming S is any non-empty set and W is the collection of all or some subsets of S whose complements are finite, then W is a topology for S and is referred to as co-finite topology. This, (S,W) is known as co-finite topological space .

Proof for co-finte topology

a) $\Phi, S \in W$ because $\Phi' = S - \Phi = S$ and $S' = S - S = \Phi$. (which is finite).

b) Suppose W_1 and $W_2 \in W$, then W_1' and W_2' are finite .

$\Rightarrow W_1' \cup W_2'$ is finite

$\Rightarrow (W_1 \cap W_2)'$ is finite (by De Morgan's law)

$\Rightarrow W_1 \cap W_2 \in W$.

c) $W_i \in W$ for all i .

$\Rightarrow W_i'$ is finite .SED

$\Rightarrow \cap\{W_i\}$ is finite .

$\Rightarrow [\cup\{W_i\}]'$ is finte.

$\Rightarrow \cup\{W_i\} \in W$.

This is known as co-finite topology

iv) The Co-countable Topology-
Assume that S is a set that is not empty and that W is either the entirety of S or all of its subsets with countable complements. W is therefore referred to as the co-countable topology for S, and (S,W) is referred to as the co-countable topological space.

v) Finer and Coarser Topology-
If S has two topologies W_1 and W_2, and $W_1 \subset W_2$,then W_2 is finer than W_1 and W_1 is coarser than W_2.

Let's use an illustration to better comprehend it.

Example:

Assume $S=\{x,y,z\}$, $W_1= \{ \Phi, S,\{x\}, \{y\},\{x,y\} \}$, $W_2= \{ \Phi, S\}$.

$\Rightarrow W_1$ is finer than W_2 and W_2 is coarser than W_1.

<u>OPEN SET</u>

Each component of W is referred to as a member of the W-open set if (S,W) is a topological space. Let's consider an illustration now.

Example:

If $S=\{p,q,r,s\}$ and $W=\{\Phi,S,\{p\},\{q,r,s\}\}$, then Φ ,S, $\{p\}$, $\{q,r,s\}$ are open set.

<u>CLOSED SET</u>

Given us a topological space (S,W). When and only when its complement is open, a subset of S is referred to as a W-closed set. Let's consider an illustration to better grasp this.

Example:

Let $S=\{a,b,c,d\}$, $W=\{\Phi,S,\{a\},\{b,c\},\{a,b,c\}\}$,
then $\Phi' = S$, $S' = \Phi$.
$\{a\}' = \{b,c,d\}$, $\{b,c\}' = \{a,d\}$, $\{a,b,c\}' = \{d\}$.
So, closed sets are Φ,S, $\{d\}$,$\{a,d\}$, $\{b,c,d\}$.

NEIGHBORHOOD

Assume that $s \in S$ and that (S,W) is a topological space. If and only if an open set G exists such that ,$s \in G \subset N$, then a subset N of S is said to be in the neighborhood of W of s.

Let's think of an illustration.

Example:

$S=\{1,2,3\}$,$W=\{\Phi,W,\{1\}\}$.
Neighborhoods of 1 are : $\{1\}$, $\{1,2\}$, $\{1,3\}$, $\{1,2,3\}$.
Neighborhood of 2 are: $\{1,2,3\}$
Neighborhoods of 3 are : $\{1,2,3\}$.

LIMIT POINT

Let A be a subset of S and (S,W) be a topological space. If every open neighborhood of s contains a point of A other than s, only then is a point $s \in S$ referred to as a limit point of A.

Now let's explore an example to attempt and grasp it.

Example:

Suppose $S=\{a,b,c,d\}$, $W=\{\Phi,S ,\{a\}, \{b,c\}, \{a,b,c,d\}\}$, then , then the limit point of $\{b,c,d\}$.

i) $a \in S$, then the open neighborhood of a are $\{a\}$, $\{a,b, c\}$,X , but $\{b,c,d\} \cap \{a\}= \Phi$. Therefore , a is not the limit point of $\{b,c,d\}$.

ii) $b \in S$, then the open neighborhoods of b are $\{b,c\}$,$\{a,b, c\}$. So , b is the limit point , since, $\{b,c,d\} \cap \{b,c\} =\{b,c\}$
$\{b,c,d\} \cap\{a,b,c\}= \{b,c\}$
$\{b,c,d\} \cap S= \{ b,c,d\}$.

iii) $c \in S$, then open neighborhoods of c are {b,c}, {a,b,c} , S . So, c is the limit point , since, $\{b,c,d\} \cap \{b,c\} = \{b,c\}$
$\{b,c,d\} \cap \{a,b,c\} = \{b,c\}$
$\{b,c,d\} \cap S = \{b,c,d\}$.

iv) $d \in S$, then the open neighborhood of d are S. So, d is the limit point , since, $\{b,c,d\} \cap S = \{b,c,d\}$.

$\Rightarrow$ b,c, and d are limit point .

NOTE: A derived set is a set of limit points.

CHAPTER 2

TOPOLOGICAL SPACE

INTERIOR POINT

In the case where (S,W) is a topological space and $I \subset S$, a point $s \in I$ is said to be an interior point of I if and only if I is in the vicinity of s, i.e., there is an open set G such that $s \in G \subset I$.

INTERIOR OF A SET

The set of all interior points of I is what is referred to as the interior of a set. I^o is used to identify this.

Let's look at an illustration

Let S={a,b, c,d} and T={Φ,S,{a}, {b,d},{a,b, d}}, then we need to find I^o , where I={b,c,d}.

Solution:

We know that the neighborhood of a are {a}, {alb},{a,c}, {a,d} , {a,b, c}, {a, c,d} , {a,b, d}, {a,b, c,d}.
Neighborhood of b are {b,d} , {b,c,d}, {a , b,d}, {a ,b,c,d } .
Neighbourhood of c is { a,b, c,d}.
Neighborhood of d is the same as that of b .
Then , I={b ,c,d} is a neighborhood of b and d both. So, I^o ={b,d} .

NOTE:

(1)The interior of I is the union of all open sets that are subsets of I.

(2) A set's interior is an open set.

EXTERIOR POINT AND EXTERIOR OF A SET

Let's assume that I belongs to a topological space S. If and only if the a point $s \in S$ is an interior point of the complement of I, it is said to be an exterior point of I. The exterior of a set defined by ext(I) is the set containing all exterior points.

Let's use an illustration to better comprehend it:

Suppose S={1,2,3,4,5} , W = {Φ, S, {1}, {2,4}, {3}, {1,3}, {1,2,4}, {2,3,4},{1,2,3,4}}. Then we need to find ext(I) where I ={1,2,5}.

Solution:

Here we know that ext(I) =(A')º.

$$\Rightarrow \text{ext}(I) = \{3,4\}.$$

$$\Rightarrow \text{ext}(I) = \Phi \cup \{3\}.$$
$$= \{3\}.$$

FRONTIER POINT AND FRONTIER OF A SET

In the event that (S,W) is a topological space, a point s is said to be a frontier point or a boundary point of a subset I of S if and only to the extent that it is neither an interior point nor an exterior point of I.
The set of all such points is then the boundary of that set.It is represented as Fr(I).

Let's consider an illustration now:
Suppose S={1,2,3,4} , W={Φ,S ,{1},{2,4},{1,2,4}}. Then we need to find Fr(I) where I ={2,3,4}.

Solution:

We know that $I^{o}=\{2,4\}$

$ext(I)= \{1\}^{o}$

$\Rightarrow ext(I)= \{1\}$.

So, 3 is neither an interior nor an exterior point of I.

CLOSURE OF A SET

Assume I⊂S and (S,W) are topological space. The intersection of all closed sets that are the superset of I determines whether a set is closed.

It is indicated as ī.

Now let's look at an illustration.

Example:

Suppose S={a,b,c,d,e} and W={Φ,S, {a}, {alb},{a,c,d}, {a,b,e}, {a,b, c,d}}. Then we need to find the closure of {c,e}.

Solution :

{Φ,S ,{b,c,d,e}, {c,d,e},{b,e},{c,d}, {e}} are the closed subset of S , then $\bar{I}$ = closure of {c,e} = {b,c,d,e} ∩ {c,d,e}∩ S = {c,d,e} .

DENSE AND NON-DENSE SET

Suppose (S,W) is a topological space and I,J ⊂ S .

Then,

(i) I is said to be dense in J if and only if J ⊂ $\bar{I}$.

(ii) I is everywhere dense in S if and only if $\bar{I}$=S.

(iii) I is no where dense in S if and only if $(\bar{I})^o$ =Φ.

(iv) I is dense in itself if and only if I ⊆ D(I).

Now let us consider an example:

Suppose S={1,2,3,4,5} and W={Φ,S,{1},{1,2} , {1,3,4},{1,2,5},{1,2,3,4}}.

Φ,S, {2,3,4,5}, {3,4,5}, {2,5}, {3,4} ,{5} are the list of closed set.

(i) Let I ={3,5} and J ={3,4} , also we know that , $\bar{I}$={ 3,4,5}.
⇒ J ⊂ $\bar{I}$ ⇒ I is dense in J .

(ii) Let I ={1,2} , then $\bar{I}$={1,2,3,4,5}=S.
⇒ I is everywhere dense in S.

(iii) Let I ={ 2,4} , then $\bar{I}$= S∩{2,3,4,5}= {2,3,4,5}. And now $(\bar{I})^o$ =Φ.
⇒ I is nowhere dense in S.

USUAL TOPOLOGY

Consider R the collection of all real numbers. U is topology on R, often known as the ordinary topology, and is defined as U = {(a, b), (c, d), (a, b) (c, d)……}. Let U be the family of subsets of R consisting of and all non-empty subset G of R having the characteristic that to any s $\in$ G.

OR

Let U be composed of and every subset G of R that is close to each s $\in$G. Regular topological space is the name given to the topological space (R, U).

Proof :

(1) Given that $\Phi \in U$ and we know that R is neighborhood of each S$\in$ R

$\Rightarrow R \in U$

(2) Let $G_1 \in U$ & $G_2 \in U$

Suppose $G_1 \cap G_2 = \Phi$, then $G_1 \cap G_2 \in U$
But if $G_1 \cap G_2 \neq \Phi$
Let $s \in G_1 \cap G_2$
$\Rightarrow s \in G_1$ and $x \in G_2$
$\Rightarrow \exists$ an open interval I_s, s.t. $x \in I_s \subseteq G_1$ and $s \in I_s \subseteq G_2$
$\Rightarrow s \in I_s \subseteq G_1 \cap G_2$
$\Rightarrow G_1 \cap G_2$ is neighborhood of each $s \in G_1 \cap G_2$
$\Rightarrow G_1 \cap G_2 \in U$

LOWER LIMIT TOPOLOGY FOR R

Assuming a collection of real numbers R and a family of all its subsets S consisting of all non-empty subsets G with the condition that each $x \in G$ a right half open interval [a, b] s.t. $x \in [a, b) \subset G$, S is referred to as R's lower limit topology.

(R, S) is a lower limit topological space .

Note:

(i) Each open interval is contained in S.

(ii) [a, b) interval that is contained in S.

(iii) $(a, b] \notin S$, since $b \in (a, b]$.

However,there does not exist any right half open interval [b, p) s.t. $b \in [b, p) \subset (a, b]$.

R'S UPPER LIMIT TOPOLOGY

Consider the set R of real numbers. H is referred to as the upper limit topology for R if it is the family of all subsets of R that includes all non-empty subset G and has the condition that to each $x \in G$ there exists a left half open interval (a, b] s.t. $x \in (a, b] \subset G$.

Upper limit topological space is the name given to the topological space (R, H).

LOCAL BASE AT A POINT OR BASE FOR A POINT'S SURROUNDING NEIGHBORHOOD SYSTEM

Given us a topological space, (X,T). Local base at x is a non-empty collection B(x) of open sets of x iff for every open set N there exist
$B \in B(x)$ s.t. $x \in B \subset N$.

Example:

Let X = {a, b, c, d, e}
T= {Φ, X, {a}, {a, b}, {a, c, d}, {a, b, c, d}}

(I) Local base at x = a
a is contained in the following open sets: {a}, {a, b}, {a, c, d}, {a, b, c, d}, X

(a) Bx = {{a}} will be the local base at a

(b) Bx = {{a}, {a, b}} will be the local base.

(c) The local basis Bx = {{a, b}} is not formed.

(II) Local base at x = b
The {a, b}, {a, b, c, d}, X are open sets that contain b

(a) Bx = {{a, b}} are local basis in b.

(b) At b, local base Bx ={{a, b, c, d}} does not form.

(III) Local base at x = c
The open sets {a, c, d}, {a, b, c, d}, X contain C

(a) Bx = {{a, c, d}} are all contained in the list.

(b) Bx = {{a, c, d}, {a, b, c, d}}

(c) Bx = {{a, b, c, d}} does not generate a local basis at position c.

IV) Local base at d
The open sets {a, c, d}, {a, b, c, d}, X. all contain the element d.
Local bases at d are equivalent to local bases at c in every way.

V) Local base at e
The only open sets that contain e are X.
Thus, the only local basis at e is X.

FIRST COUNTABLE SPACE

If every point in a topological space (X, T) has a countable local base, the space is said to be the first countable space.

Example:

If x $\in$ R and R is the set of real numbers, then the collection is {(x-(1/n), x+(1/n)); n $\in$ N} is a countable local base at x.

BASE FOR A TOPOLOGY

Given us a topological space, (X, T). If for each point x $\in$X and for each open set N including x, B $\in$B(x) s.t. x$\in$ B $\subset$N, then a non-empty collection B(x) of open set of X is said to be the base for T.

Another definition: If every open set is the union of some members of a non-empty collection of open sets B(x), then B(x) is said to be the basis for T.

Example:

Assume X = {a, b, c, d} , T= {Φ, X, {a}, {b}, {a, b}, {c, d}, {a, c, d}, {b, c, d}} and a collection of open set B(x) = {{a}, {b}, {c, d}}

(I) Open set (N) containing a is composed of the elements {a}, {a, b}, {a, c, d}, X
For each N, $\{a\} \in B(x)$ s.t. $a \in \{a\} \subset N$.

(II) {b}, {a, b), (b, c, d), X are all members of the open set (N) that contains b.
Here, $\{b\} \in B(x)$ s.t. $b \in \{b\} \subset N$ for each N.

(III) {c, d}, {a, c, d}, {b, c, d}, X are the elements of the open set (N) containing c .
Here, $\{c, d\} \in B(x)$ s.t. $c \in \{c, d\} \subset N$.

(IV) {c, d}, {a, c, d}, {b, c, d}, X are the elements of the open set (N) having d.
B(x) = a {{a}, {b}, {c. d}} so becomes the foundation for T.

SECOND COUNTABLE SET

Given us a topological space, (X, T). If it has a countable base for T, it is considered to be the second countable space.

Example:

The second countable space is (R, U).

The collection of all open intervals (r, s) with rational r and s.

As a result, each $x \in R$ and each open interval becomes N s.t. $x \in (r, s) \subset N$.

CHAPTER 3

SUBSPACE AND TOPOLOGY

SUBSPACE AND TOPOLOGY GENERATED BY SET

Take the case when (S,W) is a topological space. A family F of a subset of S is thus referred to as a sub-base of topology W if and only if $F \subset W$ and the finite intersection of members of F constitute a base for W.

Let's think about an illustration now:

Consider the following scenario: S = {a, b, c}, W={Φ,S,{a, b},{b, c},{b}}, F ={{ a, b},{b,c}}.

All finite intersection of members of F is class B . This means that B ={{b}, {a,b}, {b, c}}

Therefore, it is evident that B serves as the basis for W.
F serves as W's sub-base.

Another illustration is :

Consider the case when (R,U) is a common topology and U={collection of all open intervals}.We now take a look at another collection, F={ $(-\infty,b)$, (a,∞),....}.Now $(-\infty,11)\cap(10,\infty)=(10,11)$.

Class B is then the member F's finite intersection, meaning that it is B = {The collection of all open interval}.
A basis for U will then be B.

∴ S will serve as U's sub-base.

TOPOLOGICAL SPACE GENERATED BY SETS

S may not be a topology on S being a sub-base for unique topology on S if S is any non-empty set and F is subset of S(i.e., $F \subset S$).

Let's consider an illustration now:

For instance, S={1, 2, 3},and F={{1}, {2, 3}}. If B is made up of all finite intersections of F's members , then.i.e., B={{1},{2,3},Φ}.

Now, compute the topology, since{1} ∪ {2,3}= {1, 2, 3} = S.

As a result, W = {Φ,{1},{2, 3}, S}.

RELATIVE TOPOLOGY AND SUBSPACE

Assume S is a topological space and has topology W . Relative topology on U induced by W is the collection $W_u=\{G \cap U; G \in W\}$ is a topology on U if U is a subset of S.

Let's consider an illustration now:

Assume that U={6,7,8,9,10}, W={Φ,U,{6},{8,9},{6,8,9},{7,8,9,10}} be a topology on S and U={6,9,10} ⊂ S.

Solution:

$W_u = \{ G \cap U; G \in W \}$. So, $\Phi \cap U = \Phi$.

$S \cap U = U$

$\{6\} \cap U = \{6\}$

$\{8, 9\} \cap U = \{9\}$

$\{6, 8, 9\} \cap U = \{6, 9\}$

$\{7, 8, 9, 10\} \cap U = \{9, 10\}$

$W_u = \{\Phi, U, \{6\}, \{9\}, \{6, 9\}, \{9, 10\}$ is a topology on U.

SUBSPACE

The topological space (U, W_u) is referred to as the subspace of (S,W) if (S,W) is a topological space and W_u is a W-relative topology on U.

CONTINUITY

Think about the topological spaces (S,W) and (U,V) . If and only if there is a W-neighborhood of N of s_0 such that $f(N) \subset M$ exists for every V-neighborhood M of $f(s_0)$,a mapping $f: S \rightarrow U$, then that mapping is said to be continuous at $s_0 \in S$.

NOTE: If and only if it is continuous at each point of S, then f is said to be W-V continuous.

Let's consider an illustration now:

Take into account $S=\{1,2,3,4\}$ and $U=\{a,b,c,d\}$.
$W=\{\Phi,\{1\},\{1,2\},\{1,2,3\},S\}$ and $V=\{\Phi,\{a\},\{b\},\{a,b\},\{b,c,d\},U\}$ are topologies defined on S and U.
Let f represent a function from S to U of the form $f=\{(1,b),(2,c),(3,d),(4,c)\}$.After that, we must demonstrate that f is continuous at 4.

Solution:

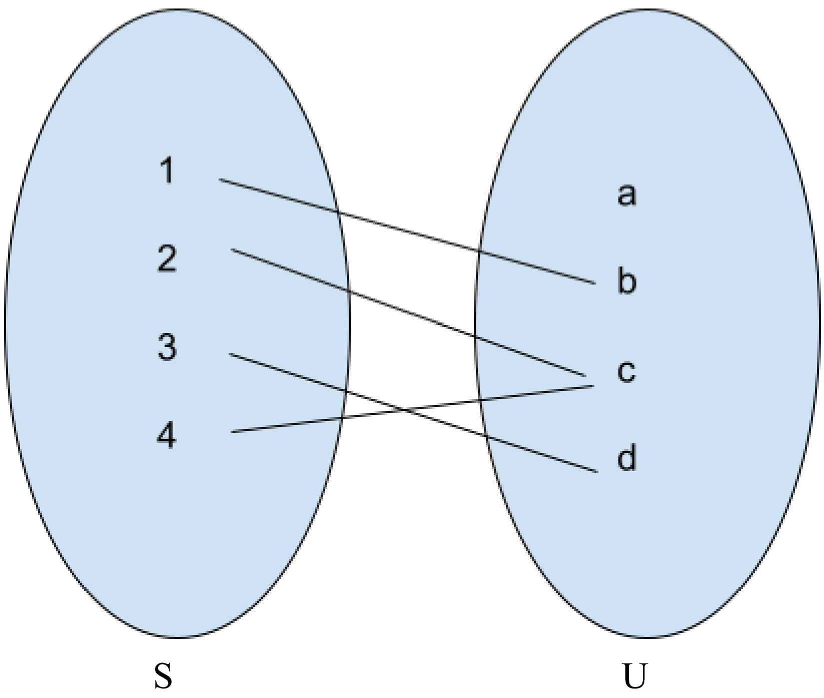
(i) f(4)=c

The list of V-neighborhoods that contain c is then comprised of b, c, and d as well as U, while S is the W-neighborhood that contains 4.

Right now, f(S)={b,c,d}.

As a result, f continues at 4.

(ii) Examine at s_0=1.



Now, f(1) =b .

List of V-nbd containing b includes {b},{a, b},{b, c, d}, U.

W-neighborhood containing 1 are {1},{1,2},{1,2,3},S.

Further , f({1})={b}

f({1,2})={b,c}

f({1,2,3})={b,c,d}

f(S)={b,c,d}

Consequently, f is continuous at 1.

RESULT:

A function f:(S,W)→(U,V) is continuous on S if and only if every V- open set has a W- open set as its inverse image.

THEOREM 1 ON CONTINUITY FUNCTION IN TOPOLOGY

A mapping f:(S,W)→(U,V) is continuous if and only if the inverse image of every V-closed subset of U is a W- closed subset of S.

Proof:

Suppose f is continuous and G be V-closed subset of U ,then we have to prove that $f^{-1}(G)$ is W-closed subset of S.

Now G' will be V-open subset of U, then $f^{-1}(G')$ will be W-open subset of S.

$\Rightarrow [f^{-1}(G)]'$ is W-open. $f^{-1}(G)$ is W-closed.

Conversely:

Let inverse image of every V-closed subset of U is W-closed subset of S. Let G be any V-open subset of U,then G' is V-closed.

$f^{-1}(G')$is W-closed.
$[f^{-1}(G)]'$ is W-closed.
$f^{-1}(G)$ is W-open.

$\Rightarrow$ f is continuous on S.

THEOREM 2 ON CONTINUITY FUNCTION IN TOPOLOGY

Suppose S and U are topological space. Then a mapping g: $(S,W)\rightarrow(U,V)$ is continuous if and only if the inverse image of every member of subbase for U is open in S.

Proof:

Suppose the inverse image of members of subbase for U is open in S.
Suppose $A^* = \{ C_1, C_2, \ldots, C_n\}$

Now, $A = C_1 \cap C_2 \cap \ldots \cap C_n\}$
Where A is a base for U and $C_1 \in V, C_2 \in V, \ldots C_n \in V$.
$\Rightarrow f^{-1}(A) = f^{-1}(C_1) \cap f^{-1}(C_2) \ldots \cap f^{-1}(C_n)$

Given that $f^{-1}(C_1), f^{-1}(C_2) \ldots, f^{-1}(C_n)$ are open in S.
diagram
$\Rightarrow f^{-1}(A)$ is open in S.

Let G be any open set in U then $G = \cup \{H; H \in A\}$
Now, $f^{-1}(G) = f^{-1}[\cup \{H; H \in A\}]$
$= \cup \{f^{-1}(H); H \in A\}$ which is open in S.

Therefore, f is continuous.

Conversely:

Suppose f is continuous then inverse image of open subset of U is open in S.
We know that members of subbase for U are open in U.

Therefore , inverse image of every members of subbase for U is open in S.

THEOREM 3 ON CONTINUITY IN TOPOLOGY

A mapping $g: S \to U$ is continuous if and only if closure of $g^{-1}[A] \subset g^{-1}[A$ closure$]$ for every $A \subset U$.

Proof:

Let g is continuous and we know that closure of A is closed in U.Then g^{-1}[closure A] is closed in U.
$\Rightarrow$closure of g^{-1}[closure A]= g^{-1}[closure A]

We also know that $A \subset$ closure A
$\Rightarrow g^{-1}(A) \subset g^{-1}$(closure A)
$\Rightarrow$closure of $g^{-1}(A) \subset$ closure of g^{-1}(closure A)
$\Rightarrow$closure of $g^{-1}(A) \subset g^{-1}$(closure A)

Conversely:

Suppose the condition hold and F be any closed subset in U, then closure of F=F.

By hypothesis ,
Closure of $g^{-1}(F) \subset g^{-1}$(closure F)
$\Rightarrow$closure of $g^{-1}(F) \subset g^{-1}(F)$ ....(1)

But we know that ,
$g^{-1}(F) \subset$ closure of $g^{-1}(F)$ ….(2) [$A \subset$ closure A]

From (1) and (2),
Closure of $g^{-1}(F)=g^{-1}(F)$
$\Rightarrow g^{-1}(F)$ is closed in S.

Therefore , g is continuous.

BICONTINUOUS MAPPING

Let's say there are two topological spaces, (S,W) and (U,V). If and only if its inverse mapping, f^{-1}: U→S, is continuous, a continuous mapping f: S→U is referred to as being bicontinuous.

A continuous mapping may also be used to describe it: f:S→U is referred to as a bicontinuous mapping if and only if the image of each W-open set of S is a V-open set of U.

Let's look at an illustration:

Assume that (S,W) and (S*,W*) are two topological spaces with S={1,2,3,4 } and S*={a,b,c, }respectively. W={Φ ,S, {1}, {2, 3} }and W* ={ Φ,S*, {a}, {b, c}}.

The mapping f: S→S* is therefore defined as f= {(1,a) ,(2, b) ,(3,c)}.

HOMEOMORPHISM

Let's say there are two topological spaces, (S,W) and (U,V). A mapping f: S→ U is called as homeomorphism if and only if ,

(i) f is one-one and onto .

(ii) f is bicontinuous.

Let's consider an illustration now:

Assume that (S,W) and (S*,W*) are two topological spaces with S={1,2,3} and S*={a, b, c } respectively. W={Φ, S, {1},{2,3}} and W* ={Φ, S*,{a},{b,c}}. f= {(1,a), (2,b), (3,c)} is a definition of a mapping f: S→S*.

This mapping, f, is both one-to-one and onto, making it bijective. The given mapping, f, is hence a homeomorphism.

NOTE:
If $f: S \rightarrow U$ is a homeomorphism and (S,W) and (U,V) are two topological spaces, then both of these spaces are homeomorphic.

OPEN COVER

If $A \subset X$ in (X,T) , then the collection of open subsets of X, $C = \{ G_\beta \}$ is said to be the open cover of A iff $A \subset \cup \{G_\beta\}$.

NOTE: C is an open cover of X if and only if $X = \cup \{G_\beta\}$.

Example 1:
Assume $X = \{1, 2, 3, 4\}$ and $T = \{\Phi, X, \{1\}, \{2\}, \{1, 2\}\}$
Let $A = \{2\} \subset X$, $C = \{\{1\}, \{1, 2\}\}$
Then $\cup C = \{1\} \cup \{1, 2\} = \{1, 2\}$
So, $A \subset \{\cup C\}$

$\Rightarrow$ C is A's open cover.

As a result, C is an open cover of all valid subsets of C.

SUBCOVER

Subcover of A is a term used to describe a subcollection C' of an open cover C of A such that C' cover A.

Example: 1

For instance, we are aware that $C' = \{1, 2\}$ is a subcollection of C and that C' is an open cover of A.
C' is hence subcover.

Example: 2

Let (R, U) be the usual topology
Suppose C = {(-n, n); n ∈ N}
Knowing that ∪ C = ∪ {(-n, n); n ∈ N} = R
So, R's open cover is C.
Since then, we have learned that C' ={(-2n, 2n); n ∈ N)} is a subcollection of C and that C' is also an open cover of R.
Then, C' is the subcover of C of R.

FINITE COVER

If an open cover of A has a finite number of members, it is said to be finite.

In example -1

C is a finite cover of A, however in case -2, C is not a finite cover of R.

BASIC AND SUBBASIC OPEN COVER

Assume that A ⊂ X and (X, T) is a topological space. If all of the members of an open cover A are contained in some base, the cover is referred to as a basic open cover.

Additionally, if all of a cover's components are contained in a single subbase, it is referred to as a sub basic open cover.

Example:

Let X = {1, 2, 3, 4} and T = {Φ, X, {1}, {2, 3}, {1, 2, 3}, {4}, {1, 4}, {2, 3, 4}} respectively.

We understand that the base of X is B = {{1}, {4}, {2, 3}}.

Consider C = {{4}, {2, 3}} and A = {2, 4}

⇒ ∪ C = {2, 3, 4}

C is the elementary open cover of {2,4} in these equations.

COMPACT SET

If each open cover in set A has a finite sub cover, then the subset is said to be compact in the topological space (X,T).

COMPACT SPACE

Every open cover of the set X of the topological space (X, T) is said to have a finite subcover iff the space is compact.

As an illustration,

Consider the usual topology (R, U), which has an infinite subcover and an open cover G = {(-n, n) | n∈ N}, is not compact.

Example:

Let T ={Φ, {a}, {b, c, d}, {c}, {a, c}, X} be the topology on X, and let X = {a, b, c, d}.Let B = {c} and G = {{a}, {c], {a, c}}.
Here, B ⊂ {∪G}

⇒ B's open cover is G.

Let G' = {{c}, {a, c}}
We are aware G' ⊂G & B ⊂ {∪G'}

⇒ G' also serves as the open cover of B.
⇒ G' is G's finite subcover.

$\Rightarrow$ Now the compact space is (X,T).

NOTE:

The space X is said to be compact iff, for every collection $\{G_\lambda: \lambda \in A)$, there are a finite number of sets $G_{\lambda 1}$,...... $G_{\lambda m}$ among the G_λ's that satisfy this, $X=G_{\lambda 1} \cup ...... \cup G_{\lambda m}$.

<u>THEOREM 1 ON COMPACT SET</u>

A continuous image of a compact space is compact.

Proof:

Let f: X $\rightarrow$Y represent a continuous mapping and (X, T) represent a compact topological space. Then to prove f[X] is compact.

Let $\{G_\lambda: \lambda \in A\}$ represent any open cover of f[X].
After that, $f[X] \subset \cup \{G_\lambda\}$.

$X \subset \cup \{f^{-1}(G_\lambda)\}$
$\Rightarrow f^{-1}(G_\lambda)$ is an open cover of X, but X is compact.

As a result, $X= f^{-1}(G_1) \cup f^{-1}(G_2) .... f^{-1}(G_n)$
$X=f^{-1} [G_1 \cup G_2 \cup \dots \cup G_n]$

$\Rightarrow f[X] = G_1 \cup G_2 \cup \dots \cup G_n$, hence f[X] is compact.

THEOREM 2 ON COMPACT SET

Closed subset of compact sets are compact.

Proof:

Assume (X, T) be a compact space, A $\subset$ X be compact, and B$\subset$ A be closed. To prove B is compact.

Let $\{G_\lambda: \lambda \in A\}$ be an open cover of B, and then B $\subset \cup \{G_\lambda\}$.
Since B is closed, X-B is open.

Consider the $\{G_\lambda: \lambda \in A\} \cup$ (X-A) family of open sets.

It is obviously an open cover of A, but provided that A is compact. As a result, this X cover has a finite subcover.

$\Rightarrow$ A = $G_1 \cup G_2 \cup .... G_n \cup$ (X-B)
$\Rightarrow$ B= $G_1 \cup G_2 \cup .... G_n$ $[A \cap (X-A) = \Phi]$
$\Rightarrow$ B is compact.

THEOREM 3 ON COMPACT SET

Illustrate that the values (0, 1) and (0, 1] are not compact.

Proof:

(1) To demonstrate that (0, 1) is not compact.

Let G = $\{(0,1-1/(n+1)); n\in N\}$ is an open cover of (0, 1)
and G'= $\{(0,1-1/(2n+1)); n\in N\}$ is a non-finite subcover of G.
As a result, (0, 1) isn't compact.

(2) To demonstrate that (0, 1] is not compact.

In this case, $G = \{ (1/n, 1]; n \in N\}$ is the open cover of (0, 1] and $G' = \{(1/2n, 1]; n \in N\}$ is the subcover of G.

$\Rightarrow$(0, 1] is not a compact value.

APPLICATIONS

Topology's low-level language, which isn't truly regarded as a distinct "branch" of topology. The study of the general abstract nature of continuity or "closeness" on spaces is known as point-set topology, sometimes known as set-theoretic topology or general topology. Continuity, dimension, compactness, and connectedness are some fundamental concepts in point-set topology. The intermediate value theorem, which states that if a path in the real line connects two integers, then it passes over each point between the two, is a basic topological conclusion. The homeomorphism between Euclidean n-space and Euclidean m-space if m=n and the maxima and minima of real valued functions on compact sets are two further arguments.

CONCLUSION

The ultimate objective of this project is to gain insight into what is known as the concept of topology in sets. A set's topology is a collection of subsets that encompasses certainly the empty set and the true set, along with closed unions and finite intersections. The topology's sets are open, and their counterparts are closed. Some crucial topologies in sets definitions are presented in the first chapter. The subsequent chapter covers interior points, exterior points, frontier points, a topological proof, and a few definitions. We encounter certain theorems relating to continuity and compact set in the third chapter.The ultimate objective of this project is to gain insight into what is known as the concept of topology in sets. A set's topology is a collection of subsets that encompasses certainly the empty set and the true set, along with closed unions and finite intersections. The topology's sets are open, and their counterparts are closed. Some crucial topologies in sets definitions are presented in the first chapter. The subsequent chapter covers interior points, exterior points, frontier points, a topological proof, and a few definitions. We encounter certain theorems relating to continuity and compact set in the third chapter.The research done for this project enabled us to gain greater comprehension of set topology.

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