CLASSIFYING GROUPS OF ORDER UPTO 15

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DECLARATION

We here by declare that this project entitled **"CLASSIFYING GOUPS OF ORDER UPTO 15"** is a bonafide work done by us under the supervision of Dr. SEETHU VARGHESE, Head of the Department of Mathematics, Bharata Mata College, Thrikkakara and the work has not previously formed by the basis for the award of any academic qualification, fellowship or the other similarity of any other University or Board.

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CERTIFICATE

This is to certify that the project entitled **"CLASSIFYING GROUPS OF ORDER UPTO 15"** submitted jointly by Jubna A P, Akhil Jojo,

Anamika K A, Revathi C B, K H Aparna and Swaroop Shankar K S in partial fulfilment of the requirements for the B.Sc. Degree in Mathematics is a bonafide record of the studies undertaken by them under my supervision at the Department of Mathematics, Bharata Mata College, Thrikkakara, during 2021 – 2022. This dissertation has not been submitted for any other degree elsewhere.

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INTRODUCTION

This project is the study of classification of groups of order less than or equal to 15 based on the theorems like Sylow theorem, Cauchy's theorem, Lagrange theorem etc. This is done by classifying groups into order of prime p, 2p, p^2, p^3, pq, p^2q. This project aims to achieve the classification as simply as possible in a way which can be easily incorporated into a first course in abstract algebra. The method which is used here can be applied to classifying groups of higher orders are the highlights.

Chapter 1

PRELIMINARIES

1.BINARY OPERATIONS

A binary operation on a set is a rule that assigns to each ordered pair of elements of the set to some element of the set.i.e., a binary operation on a set S is a function mapping from SxS into S.

If * is a binary operation on the set S we say that S is closed under the operation *.

If non empty set S is closed under the operation * , then a * b ∈ S for a,b ∈ S.

2.GROUPS

A group (G,*) is a set G , closed under a binary operation * , such that the following axioms are satisfied.

Associativity

The binary operation * is associative.

i.e.(a * b) * c = a*(b*c) for every a , b , c ∈ G.

Existence of identity

There is an element in G such that e * a = a*e = a for every a ∈ G.

Existence of inverse

For each a ∈ G, there is an element a^{-1} in G such that

$a^{-1} * a = a * a^{-1} = e$

3.ABELIAN GROUP

A group (G , *) is said to be abelian or commutative if the following axioms holds.

Commutativity : a* b = b*a for every a , b ∈ G.

Example : a set of integers is a group with respect to the operation of addition of integers.

4.NON ABELIAN GROUP

Group (G, ∗) in which there exists at least one pair of elements a and b of G, such that $a * b \neq b * a$.

5.SUBGROUP

If a subset of a group G is closed under the binary operation of G and if H with the induced operation from G is itself is a group , then H is a subgroup of G . We denote it as

$H \leq G$

Example

- (z , +) is a subgroup of (R , +)
- (Q , +) is a subgroup of (R ,+)

6.CYCLIC SUBGROUP

Let G be a group and let a € G . Then , H = $\{a^n : n \in Z\}$ is the cyclic subgroup of G generated by ‘ a’ . Cyclic subgroup of G generated by ‘a’ is denoted by <a>.

7.HOMOMORPHISM OF GROUPS

Let (G,o) & (G’,o’) be 2 groups, a mapping “f ” from a group (G,o) to a group (G’,o’) is said to be a homomorphism if

$f(aob) = f(a)\ o'\ f(b)\ \forall\ a,b \in G$

The essential point here is : The mapping f : G –> G’ may neither be a one-one nor onto mapping, i.e, ‘f’ needs not to be bijective

8.ISOMORPHISM

Two groups are said to be isomorphic if there exist bijective homomorphism between them.

9.ORDER OF A GROUP AND ORDER OF AN ELEMENT

The order of a group (G) is the number of elements present in that group, i.e it's cardinality. It is denoted by |G|. Order of element a ∈ G is the smallest positive integer n, such that a^n= e, where e denotes the identity element of the group, and an denotes the product of n copies of a.

10.COSETS

Let H be a subgroup of group G , and let a ∈ G . The left coset aH of H is the set { ah/ h ∈ H} . That is ,

aH = {ah / h ∈ H }

The right coset Ha of H is the set {ha/h ∈ H } . That is ,

$Ha = \{ha / h \in H\}$

11.LAGRANGE'S THEOREM

Let H be a subgroup of finite group G . Then the order of H is a divisor of order of G. I.e O(H) divides O(G).

12.INDEX OF H IN G

Let H be a subgroup of G . The number of left left cosets of H in G is the index (G:H) of H in G

(G:H) = O (G)/ O(H)

13.NORMAL SUBGROUPS

A subgroup H of a group G is a normal subgroup of G if $ghg^{-1} \in H$ for all $g \in G$.

14.TRIVIAL GROUP

A group consisting of single element.

15.DIHEDRAL GROUP

Dihedral group is the group of symmetries of a regular polygon, which includes rotations and reflections. Dihedral groups are among the simplest examples of finite groups.

A dihedral group

$D_{2n} = \langle x, y \mid x^n = 1, y^2 = 1, yxy = x^{-1} \rangle$

16.CAUCHYS THEOREM

Cauchy's theorem states that if G is a finite group and p is a prime number dividing the order of G (the number of elements in G), then G contains an element of order p.

17.QUATERNION GROUP Q_8

The quaternion group is one of the two non abelian groups of the five finite groups of order 8

$Q_8 = < x,y \mid x^4 = 1, x^2 = y^2, yxy^{-1} = x^{-1} >$

18.FIRST SYLOW THEOREM

Let G be a finite group and p a prime number. If p divides the order of G, then G has a subgroup of order p.

19.SYLOW p-SUBGROUP

If p^k is the highest power of a prime p dividing the order of a finite group G, then a subgroup of G of order p^k is called a Sylow p-subgroup.

20.SECOND SYLOW THEOREM

Let P be a Sylow p-subgroup of the finite group G. Let Q be any p-subgroup of G. Then Q is a subset of a conjugate of P.

21.THIRD SYLOW THEOREM

Any two p-Sylow subgroups of a finite group G are conjugate. Moreover if P is the number of distinct p-Sylow subgroups of G, then P divides |G| and P = pt + 1 for some integer t ≥ 0.

22.KERNEL

The kernel is the set of all elements in G which map to the identity element in H.

23.SECOND ISOMORPHISM THEOREM

Let G be a group, let $H \leq G$ and let $N \trianglelefteq G$. Then the set

$HN = \{ hn : h \in H, n \in N \}$

is a subgroup of G, $H \cap N \trianglelefteq H$, and $H/(H \cap N) \simeq HN/N$.

where $\trianglelefteq$ means normal subgroup of or equal to.

Chapter 2

GROUPS OF ORDER PRIME p AND 2p

Proposition 2.1

There is only one group of order prime upto isomorphism

Lemma 2.1.1 :

Every group of prime order is cyclic

Proof :

Let p be a prime and G be a group such that |G| = p. Then G contains more than one element. Let g ε G such that g ≠ eG . Then

<g> contains more than one element. Since <g> ≤ G, by Lagrange's theorem, |<g>| divides p. Since |<g>| > 1 and |<g>| divides a prime, |<g>| = p = |G|. Hence, <g> = G. It follows that G is cyclic.

Lemma 2.1.2 :

Any finite cyclic group is isomorphic to Z_n

Proof :

Let Φ be a function from Z or Z_n to <a>

i.e, $\Phi(K) = a^K$

$\Phi(K1 + K2) = a^{K1} a^{K2} = \Phi(K_1)\,\Phi(K_2)$

$\Phi(K_1) = \Phi(K_2) \Rightarrow a^{K1} = a^{K2} \Rightarrow a^{K1-K2} = e$

When <a> is finite , this is only possible when $K_1 = K_2$. When <a> has order n, this means $n|(K_1 - K_2)$. This means $K_1 = K_2$ because $K_1, K_2 \in Z_n$. Therefore, Φ is one-to-one.

Every element in <a> can be written as ak = Φ (K). When <a> has finite order, K ∈ Z. Therefore, Φ is onto.

Therefore, Φ is an isomorphism from Z or Z_n to <a>

Result :

The consequence of this is that the group of orders

1, 2, 3, 5, 7, 11, 13 …. have only one group upto isomorphism

For n=1 , the trivial group <e> = {e}

For n = 2, 3 , 5, 7, 11, 13

$Z_2, Z_3, Z_5, Z_7, Z_{11}, Z_{13}$

Proposition 2.3 :

There are exactly two isomorphism classes of group of order 2p where p is prime.

Lemma 2.3.1:

If p is an odd prime, then every group of order 2p is isomorphic either to the cyclic group Z_{2p} or the dihedral group D_p of order 2p

Proof:

Let G be a group of order 2p, where p is an odd prime.

By Cauchy's theorem, G has an element a of order p and b of order 2.

Let H=(a)={e,a,a^2,..........p-1}.

Since H does not contain any element of 2, b € H.

Since {G:H} =2 ⇒ H is normal in G.

Therefore , $bab^{-1} \in H$.

Since $a \neq e \Rightarrow bab^{-1} \neq e$.

Hence , $bab^{-1} = a^t$ for some t with $1 \leq t \leq p-1$.

Then as o(b) = 2,

$A = b^2ab^{-2} = b(bab^{-1})b^{-1} = ba^tb^{-1} = (bab^{-1})^t = (a^t)^t = a^{t^2}$

$\Rightarrow a^{t^2-1} = e$

But 0(a) = p ⇒p divides $t^2-1 = (t-1)(t+1)$.

Hence p divides t-1 or t+1.

This is possible only if t=1 or t= p-1.

Result :

Thus there are two non isomorphic groups of order 6, one is abelian and the other is non abelian viz, Z_6 and D_3.

Thus there two isomorphic group of order 10, one is abelian and the other is non abelian, viz. Z_{10} and D_5.

Similarly there are two non isomorphic group of order 14, one is abelian and the other is non abelian ,viz. Z_{14} and D_7.

Chapter 3

GROUPS OF ORDER PRIME SQUARE (p^2) AND PRIME CUBE (p^3)

Proposition 3.1 :

There are only two groups of order p^2, where p is a prime.

Lemma 3.1.1 :

If p is a prime number, then any group of order p^2 is abelian.

Proof :

Let G be a group and $O(G)=p^2$

and Z(G) be the centre of group.

So we need to show that G=Z(G).

Since Z(G) is a subgroup of G

$\Rightarrow |Z(G)| / |G|$

$\Rightarrow O(Z(G)) = p$ or p^2

If $O(G) = p^2 \Rightarrow Z(G) = G$

Hence G is abelian

So let $O(G) = p$

Let $a \in G$, $a \notin Z(G)$

$\exists$ $N(a)$ is a subgroup of G

$Z(G) \leq N(a)$

Let $x \in Z(G)$

$\Rightarrow xy = yx \;\; \forall\, y \in G$

$\Rightarrow xa = ax \; \forall\, a \in G$

$\Rightarrow x \in N(a)$

$\Rightarrow Z(G) < N(a)$

$\Rightarrow O(Z(G)) < O(N(a))$

$a \notin Z(G)$

$\Rightarrow a \in N(a)$

But N(a) is subgroup of G.

$\Rightarrow O(N(a)) \,/\, O(G)$

$\Rightarrow O(N(a)) = p^2$

$\Rightarrow N(a) = G$

$\Rightarrow a \in Z(G)$

which is a contradiction

$\Rightarrow O(Z(G)) \neq p$

$\Rightarrow O(Z(G)) = p^2$

$\Rightarrow Z(G) = G$

Hence G is abelian.

Lemma 3.1.2 :

Let p be prime there are only two groups upto isomorphism of order p^2

Proof :

Suppose G is a p^2 group. It is abelian.

According to Lagrange Theorem, the divisors of p^2 are 1, p, p^2.

Let x has order p^2 then $G = <x>$ generates all the elements of the group $G = p^2$

So G is cyclic. This satisfies the earlier notation that G is abelian.

$G \cong Z_{p^2}$

Now assume that there is no element of order p^2.

This means that every element which is not the identity has order p. Pick x order p. Since $< x > \leq G$, you can take another order p element y in the complement of $< x >$.

Now

Θ: (u, v) → uv

yields a homomorphism from <x> × <y> to G.

Note that <x> n <y> = <e>, so the latter is injective. Since by Lagrange theorem both groups have same cardinality, it follows that Θ is an isomorphism. If < y > is a complement of < x > it suffices that only the identity element will be the intersection since they are different primes. And of course we all know that the cardinality of primes is always the same. It implies that Θ is an isomorphism.

Finally since $< x > \cong < y > \cong Z_p$

$G \cong < x > \times < y > \cong Z_p \times Z_p$

So G is either isomorphic to Z_p

2 or to $Z_p \times Z_p$ of course the implication of this is that every group of p^2 is either Z_{p^2} or $Z_p \times Z_p$.

i.e. there are only two groups of order p^2 up to isomorphism.

This covers groups of order 4, 9…

Proposition 3.2

There are five groups of order p^3 either

1. $G \cong Z_{p^3} \cong Z_p \times Z_{p^2} \cong Z_p \times Z_p \times Z_p$ Or
2. $G \cong D_{p^3} \cong Q_{p^3}$

Proof

From the proposition above (3.2) we can deduce that by

transitive property that

$G \cong Z_{p^3}$

$G \cong Z_P \times Z_{P^2}$

$G \cong Z_P \times Z_P \times Z_P$

$G \cong D_{p^3}$

$G \cong Q_{p^3}$

That's five groups in total

by disjunctive syllogism i.e. either / or,

Suppose G is not cyclic

by Lagrange theorem

only divisors of p^3

1, p, p^2 , p^3

But we can't take order p^3 because

$< x > = p^3$ will generate all the members of the group

making it cyclic. So we have 1, p, p^2.

Suppose we take $|b| = p$; $< b >$ will generate all the

members of p and suppose we take $|a| = p^2$; $< a >$ will

generate all the members of p^2. Hence group of order p^3

must contain some cyclic groups.

But let order p^3 have x, y & z; recall $G = Z_{p^3}$

hence $|< x >| \leq G$ and $|< y >| \leq G$ also $|< z >| \leq G$

$F: (x, y, z) \longrightarrow x \times y \times z$

Let F be a homomorphism that map $< x > \times < y > \times < z >$ to G

Of course since $\{x, y, z\} \in p^2$ and also {x, y, z} is contained in p^2.

Then $< x > \cap < y > \cap < z > = e$ and they must have the same cardinality iff {x, y, z} are subgroups of order p^2 and are

contained in G i.e. $G = p^3$

then $G \cong < x > \cong < y > \cong < z > \cong Z_{p^3}$

$G \cong < x > \times < y > \times < z > \cong Z_{p^2} \times Z_P$

$G \cong < x > \times < y > \times < z > \cong Z_P \times Z_P \times Z_P$

But if $|b| = p$ & $|a| = p^2$; then b = 2

Hence b is an evolution; therefore being its own inverse

$ab = (ab)^{-1} = b^{-1}a^{-1}$

But b is an evolution

$\Rightarrow ab = ba^{-1}$ and from the dihedral group we know that

$D_{2n} = < a, b / a^{n} = y^{2} = 1; ab = ba^{-1}>$

And also of the Quaternion group

$Q_{4n} = < a, b / a^{2n} = y^{4} = 1; ab = ba^{-1}>$

the same way a, b relate to the cyclic group of order p^3, is

the same way a, b relate to the dihedral and Quaternion

groups of same order.

G ⇒ dihedral G ⇒ Quartenion & G ⇒ cyclic

This covers groups of order 8.

Chapter 4

GROUPS OF ORDER pq AND p^2q

Proposition 4.1

If G is a group of order pq for some primes p, q such that p>q and q doesn't divide (p-1) then

$$G \cong Z_{pq} \cong Z_p \times Z_q$$

Proof :

We can find a unique sylow p and sylow q subgroups of G.

By the third sylow theorem,

Let S_q be sylow q & S_p be sylow p

$S_p |$ q and $S_p = 1 + kp$

Since q is a prime the first condition gives $S_p = 1$ or $S_q = q$

Since p>q the second condition implies then that $S_p = 1$.

Similarly let S_q be the number of sylow q- subgroups of G.

We have

S_q | p and $S_q = 1+ kq$

the first condition gives $S_q = 1$ or $S_q = p$.

If $S_q = p$ then the first second condition gives p = 1 + kq, or p - 1 = kq

This is however impossible since q doesn't divide (p-1).

Therefore, we have $S_q = 1$.

It means we have a unique sylow p subgroup and a unique sylow q subgroup. By the second law of Sylow's theorem, every element of G of order p belongs to the subgroup p and every element of order q belongs to the subgroup q. It follows that G contains exactly p -1 elements of order p, exactly q-1 elements of order q and one trivial element of order 1. Since for p,q we have

$$pq > (p-1) + (q-1) + 1$$

There are elements of G of order not equal to 1, p or q. Any such element must have order pq.

We can assume an element x of order p and y of order q: y is a complement of x

$|<x>| \leq G$ and $|<y>| \leq G$

F: $(x, y) \rightarrow x \times y$

Let F be a homomorphism from $<x> \times <y>$ to G,

We have the right to do that since $<x>$ n $<y>= \{e\}$. By Lagrange theorem, the divisors of prime (p) are {1 and p}, hence it follows that $|x|$ and $|y|$ have the same cardinality. It suffices that F is an Isomorphism

$<x> \cong <y> \cong Z_{pq}$

$G \cong <x> \times <y> \cong Z_p \times Z_q$

this covers groups of order 15...

Corollary 4.1.1:

Every group of Z_{pq} is isomorphic to $Z_p \times Z_q$ and there is only one order pq.

Proposition 4.2

For every Abelian group of order p^2q;

(i) $G \cong Z_{p^2} \times Z_q$

(ii) $G \cong Z_p \times Z_p \times Z_q$

Proof :

Suppose G is a finite group of order p^2q for all p, q distinct primes p^2 is not congruent to 1 mod p and q is not congruent to1 mod p, then G is Abelian.

By sylow's theorem,

np = 1+ kp and it must divide p^2q.

So, 1+ kp/q and because q is not congruent to 1 mod p

$\Rightarrow$ np = 1.

This means we have a unique sylow p (G) for an example of p in the group and is normal and also Isomorphic to Z_{p^2} or $Z_p \times Z_p$.

Since q does not divide p^2 - 1, therefore nq = 1+ kq is not congruent to p, p^2. So we also have a normal sylow q (G).

Hence G is Abelian,

$G \cong Z_{p^2} \times Z_q$

$G \cong Z_p \times Z_p \times Z_q$

Another simpler way to see this is;

$np^2/q = 1 \bmod p^2 = \{1, p, p^2\}$

$nq/p^2 = 1 \bmod q = \{1, q) = 1$

Hence sylow q (G) is characteristically normal in G, that is we have a unique sylow q (G).

Let order p^2q have x^2, y. $\forall$ G = p^2q, and let $x^2 \in p^2$ and $y \in q$,

$|< x^2 >| \leq G$, also $| < y > | \leq G$

Suppose $\theta : x^2, y \rightarrow x^2 \times y$ is a homomorphism that maps x^2, y to G.

Since y has order prime (q), and p and q are distinct;

$x^2 \cap y = \{e\}$ and $| x2 | = | y |$, hence $x^2 \cong |y| \cong x \times x \times y$

$G \cong x^2 \times y$

$G \cong x \times x \times y$

Hence; $G \cong Z_{p^2} \times Z_q$

$G \cong Z_p \times Z_p \times Z_q$

This covers abelian groups of order 12.

Corollary 4.2.1 :

There are only two abelian groups of order p^2q, upto isomorphism.

Proposition 4.3

There are exactly three non abelian group of order 12.

Proof:

First suppose that Sylow 3-subgroup of G is normal. By Cauchy's theorem, G has an element a of order 3. Then $\langle a \rangle \lhd G$. Let A be a Sylow

2-subgroup of G. Then $|A| = 4$ $(= 2^2)$, hence abelian.

Thus, either $A \simeq Z_2 \oplus Z_2$ or $A \simeq Z_4$, and $G = \langle a \rangle A$.

First assume that $A \simeq Z_2 \oplus Z_2$. If every element of A commutes with a, then G is abelian. Thus, there is at least one element of A which does not commute with a.

Since $A \simeq Z_2 \oplus Z_2$, so nonidentity elements of A are u, v and uv.
Without any loss of generality we can assume that $uau \neq a$.
Thus, $uau = a^{-1}$. If $va \neq av$, then $vav = a^{-1}$, and so $uvauv = a$,
that is uv commutes with a.
If $uvauv = a^{-1}$, then $vav = u(uvauv)u = u(a^{-1})u = a$.
Thus there is exactly one element of A which commutes with a.
Let it be v. Then $x = av$, is of order 6.
Let $y \in A$ such that y does not commute with a. Then $yay = a^{-1}$.

Now $yxy = yayyvy = a^{-1}v = (av)^{-1} = x^{-1}$.

Thus, $G = \langle x, y \mid x^6 = 1, y^2 = 1, yxy = x^{-1}\rangle$, a dihedral group of order 12.

Let $A \simeq Z_4$. Then $A = \langle y\rangle$ for some $y \in G$ and a and y will not commute; otherwise G will be abelian.

Since $o(a) = 3$, so $yay^{-1} = a^{-1}$. Thus, $y^2ay^{-2} = a$ that is, y^2 commutes with a.

Let $x=ay^2$. Then $o(x) = 6$ and $x^3 = a^3 y^6= y^2$.

Finally

$(xy)^2 = xyxy = (ay^2)y(ay^2)y= (ay^3) (ay^3) = y^3(yay^{-1})ay^{-1} = y^3(a^{-1}a)y^{-1} = y^2$.

Thus, G is isomorphic to T.

Finally, we assume that Sylow 3-subgroup is not normal in G.
Let P be a Sylow 3-subgroup of G. Then $|G: P| = 4$ and there is a homomorphism $\Psi: G \to S_4$ such that $\ker \Psi \le P$.
Since $|P|= 3$, so $\ker\Psi = \{1\}$ or P. If $\ker\Psi = P$, then $P \lhd G$, which is not the case, and so $\ker\Psi = \{1\}$. Hence, $\Psi: G \to S_4$ is a monomorphism.
Thus, $\Psi(G)$ is a subgroup of S_4 of order 12.
If $\Psi(G) \neq A_4$, then $\Psi(G) A_4 \le S_4$ and $|\Psi(G) A_4/A_4| = 2$, by second isomorphism theorem $|A_4/\Psi(G) \cap A_4|= 2$ a contradiction as A_4 has no subgroups of order 6.

Thus, $\Psi(G) = A_4$, and so $G \simeq A_4$.

Result:

There are five isomorphism classes of groups of order 12.

$Z_4 \times Z_3$

$Z_2 \times Z_2 \times Z_3$

A_4

D_{12}

$T = (x,y \mid x^6 = 1, y^2 = x^3 = (xy)^3)$

CONCLUSION

Here we classified groups of order less than or equal to 15. We proved that there is only one group of order prime up to isomorphism, and that all groups of order prime (p) are abelian groups. This covers groups of order 2,3,5,7,11,13. Again we were able to prove that there are up to isomorphism only two groups of order 2p, where p is prime and $p \geq 3$, and this is $Z_{2p} \cong Z_2 \times Z_P$ (where Z represents cyclic group) and D_P (the dihedral group of the p-gon). This covers groups of order 6, 10, 14 and we proved that up to isomorphism there are only two groups of order p^2 and these are Z_{p^2} and $Z_p \times Z_p$. This covers groups of order 4, 9. Groups of order p^3 was also dealt with, and we proved that there are up to isomorphism five groups of order P^3 which are Z_{p^3}, $Z_{p^2} \times Z_{p,}$ $Z_P \times Z_P \times Z_{P,}$ D_{p^3} and Q_{p^3}. This covers for groups of order 8. Sylow's theorem was used to classify groups of order pq, where p and q are two distinct primes.

And there is only one group of such order up to isomorphism, which is $Z_{pq} \cong Z_p \times Z_q$. This covers groups of order 15. Sylow's theorem was also used to classify groups of order p^2q and t here are only two abelian groups of such order which are Z_{p^2q} and $Z_p \times Z_p \times Z_q$. This covers order 12. Finally groups of order one are the trivial groups. And all groups of order 1 are abelian because the trivial subgroup of any group is a normal subgroup of that group.

CLASSIFICATION OF GROUPS OF ORDER UPTO 15

ORDER	GROUP
1	Z_1
2	Z_2
3	Z_3
4	Z_4 Klein 4 – group V $\cong Z_2 \times Z_2$
5	Z_5
6	$Z_6 \cong Z_2 \times Z_3$ D_3
7	Z_7
8	Z_8 $Z_2 \times Z_4$ $Z_2 \times Z_2 \times Z_2$ D_4 Q_8
9	Z_9 $Z_3 \times Z_3$
10	$Z_{10} \cong Z_2 \times Z_5$ D_5
11	Z_{11}
12	$Z_{12} \cong Z_3 \times Z_4$

	$Z_2 Z_6 \cong Z_2 \times Z_2 \times Z_3$ D_6 A_4 T
13	Z_{13}
14	$Z_{14} \cong Z_2 \times Z_7$ D_7
15	$Z_{15} \cong Z_3 \times Z_5$

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