CRIMINAL INVESTIGATION THROUGH MATHEMATICAL EXAMINATIONS

Project Report Submitted to

Mahatma Gandhi University

In partial fulfillment of the requirement for the award of the degree of

MASTER OF SCIENCE IN MATHEMATICS

Submitted By

SANJAY KUMAR V S

Reg No. 200011014784

Under The Supervision of

Dr. Seethu Varghese

DEPARTMENT OF MATHEMATICS

Bharata Mata College, Thrikkakara

2020-2022

CERTIFICATE

This is to certify that the dissertation entitled **"CRIMINAL INVESTIGATION THROUGH MATHEMATICAL EXAMINATIONS"** submitted by SANJAY KUMAR V S is a record of work done by the candidate during the period of her study under my supervision and guidance.

Dr. Seethu Varghese

Department of Mathematics

Bharata Mata College,

Thrikkakara

Place: Thirikkakara

Date:

DECLARATION

I hereby that the project report entitled **"CRIMINAL INVESTIGATION THROUGH MATHEMATICAL EXAMINATIONS"** Submitted for the MSc. Degree is my original work done under the supervision of Dr. **SEETHU VARGHESE** and the project has not formed the basis for the award of my academic qualification fellowship or other similar title of other university or board

Place:

Date **:** **SANJAY KUMAR V S**

ACKNOWLEDGEMENT

I sincerely thank each and every one who has helped me for the successful completion of this project. I thank God Almighty for all his blessings showered upon me. I would like to place on record all my gratitude to Dr. Seethu Varghese, Department of Mathematics, Bharata Mata College, Thrikkakara, who has provided her valuable guidance throughout this project work.

Also, I thank all other teachers of the department of their encouragement. My friends were also helpful to me through their suggestions for which I am grateful towards them

SANJAY KUMAR V S

Place: Thrikkakara

Date:

CONTENTS

ABSTRACT

Mathematics is fast becoming one of the most important techniques in crime detection. In the first chapter of this project discuss the importance of mathematics in forensic science. Also discuss the different branches of mathematics like calculus, measurements, proportion, trigonometry, probability are used in criminal investigation. In the second chapter discuss trigonometric methods in forensic science like Pythagoras theorem, trigonometric functions etc. By the use of trigonometry discuss different types of blood spatter, finding angle of impact of blood spatter, area of convergence and determine the height of blood fell. In the third chapter is left to discuss the determination of estimated time of death using differential equations. Using newton's law of cooling and examples to find the estimated time of death of a person. In the fourth chapter discuss application of graph theory in criminal investigation. Here we analysis some definitions of graph colouring and some prepositions to finding a thief using graph colouring technique. In the last chapter discuss the use of probability in forensic science. Human teeth marks, matching of hair evidence and finding the location of criminal living are also discussed in this chapter.

INTRODUCTION

Forensic science is any branch of science used to analyse crime scene evidence for a court of law. All science uses maths concepts and equations and forensic scientist are well educated in mathematical concepts they use to analyse evidence from crime scene.

The main things crime scene investigators do is collect measures and document evidence. Their data helps forensic scientist perform calculation and determine the facts of a crime. Math makes it possible to show proof of what occurred during a crime in data and number.

The court asks it is impossible to analyse forensic evidence without maths. One of about the significance of evidence in context of the crime and as an expert witness. The forensic scientist should be able to respond appropriately to such a challenge. Methods based on Bayesian statistics utilizing probabilistic arguments may facilitate both of the types and the weight that should be attached to each by court.

The discussions and presentation of any quantitative data within the report submitted to the court by the expert witness must be prepared with rigour and clarity that can only come from a sound understanding of the essential mathematical and statistical methods applied within forensic science.

CHAPTER 1
MATHEMATICS IN FORENSIC SCIENCE

Forensic science is the application of science to criminal and civil laws mainly on the criminal side during criminal investigation as governed by the legal standards of admissible evidence and criminal procedure.

Forensic science is an ever-growing field that can be further subdivided in to Toxicology, Anthropology, and Odontology. Mathematics and many other applications of calculus of forensic science can be most clearly being seen in the fields of forensic biology and pathology. For example, to put it in a simpler terms power series are to functions what DNA molecules are to people.

Specifically for pathologists, calculus is needed to estimate the time of death of victims. Overall calculus has many application to many of the subfield and forensics and is often a useful tool in crime scene investigation Forensic scientists collect preserve and analyse scientific evidence themselves. Others occupy a laboratory role performing analysis on objects brought to them by other individuals

MEASUREMENT

One area of maths is crucial to forensic science is taking precise measurements at a crime scene knowing the exact length of a shoe print could whose shoes are the wrong size. For example, forensic scientists need exact measurement of everything at a crime scene in order to perform scientific calculations properly. Investigators spend a great deal of time measuring distance, weight temperature volume and other aspects of evidence to get the number correct.

PROPOTIONS

Forensic scientists use not only measurements but proportion in other analysis. If a human leg bone is discovered in a unmarked grave, for example forensic scientists use math equations to determine what proportion or percentage of a person overall height the leg bone would be. Once they know that they can determine how tall the person was and whether it was a child or adult proportions are one way math is involved in forensic science

APPLICATION OF DIFFERENT BRANCH OF MATHS IN CRIMINAL INVESTIGATION

Forensic scientists analyse the existence in and around crime scenes for clues pointing to possible suspects causes of death or time of death. Math can be used to determine how crime are committed when they were committed and who committed and who committed them

- Psycho physical detection –Monitoring plus rate, blood pressure, and breathing patterns
- Heights and distance – Footprints in dirt and mud length of strides
- Entomology-time of death
- Trigonometry and industrial physics can be used to reverse calculate height
- Examining the skid mark can help to reconstruct the accident mark that are caused by the speed of the car, breaking forces
- Newton's law of cooling describes the cooling of a warmer objects to the cooler temperature of its environment

CALCULUS IN FORENSIC SCIENCE

A forensic analyst uses blood stain pattern analysis in order to tell the story of the crime. It turns out that the location where the blood lands and the shape of a blood on the landing surface reveal both the direction in which the blood was used to wound the victim,

Analysis use math principles to figure out the location of the victim where the blood was shed and even the type of weapons or impact that caused the victims injury. Sometimes from a weapon can even reveal a criminals mentality

A different kind of calculus is used by medical examiners and forensic technicians ever day to estimate time of death of a victim. When a victim dies they experience three stages of death, where the body starts to decompose

Algor mortis is the rate at which the body cools after death. Algor mortis is used to estimate time of death, importantly, if the body is discovered only a few hours or less after death, to determine the time of death by using Newton's law of cooling

$\frac{dT}{dt}$ =K (T-T_0) where T is the temperature of the object, T_0 is the temperature of the Surroundings.

PROBABILITY

Random math probabilities are used to estimate and express the rarity of a DNA that someone else in the population chosen at random would have the same genotype as the genotype of the contributor of the forensic evidence. RMP is calculated during the genotype frequencies at all the loci or how common or rare the alleles of a genotype are RMP can only be used as a statistic to describe the DNA profile if it is from a single source or if the analysis is able to differentiate between the peaks on the electropherogram from the major and minor contributors of a mixture.

Combined probability of inclusion is a common statistic used when the analyst cannot differentiate between the peaks from the major and minor contributor to a sample and the number of contributors cannot be determined

TRIGNOMETRY

Trigonometry is very useful in forensic science. Knowledge of trigonometry is absolutely necessary for many crime scene reconstructions blood stain pattern analysis is one of the several specialists in the field of forensic science involves the study and analysis of blood stains at a known or suspected violent acts such as assault, homicide, abduction, suicide or even vehicular accidents.

Pythagoras theorem, Trigonometry function, Trigonometry rules are applications of trigonometry in forensic science. Trigonometry functions relate to non-right angled triangles and can be used to find a uniform angle or side.

POLICE MATHEMATICS

Police use mathematics every day on their job to solve crimes. Policemen jobsare to find out what happened at the sight of the scene of the crime or accident. They use math on the job to explain data that needs to be stored for information.

For example, a speeding car causing skid marks. Data can be stored and interpreted using wavelets, probability and statistics. It can be securely transmitted using prime numbers and cryptography. But first of all, police must get at the information underlying the data. They must look at all the evidence left at the crime scene and work backwards to deduce what happened and who did it. In order for the officer to find out how fast the car was going at the scene he needs to solve an inverse problem. Inverse problems are mathematical detective problems. An example of an inverse problem is trying to find the shape of an object only knowing its shadows. In addition, a day on the job of being a cop. There is a car accident and the officer job is to figure out if the car was speeding or not. The only information the police officer has is the damage to the car, witnesses, and skid marks from the vehicle. By investigating the skid marks can help rebuild the accident. The marks are caused by the speed of the car as well as other factors such as braking force, friction with the road and impacts with other vehicles. Without any witnesses at the scene of the accident, the police officers have to plug in a formula to figure out who is at fault of the accident.

CHAPTER 2

TRIGONOMETRIC METHODS IN FORENSIC SCIENCE

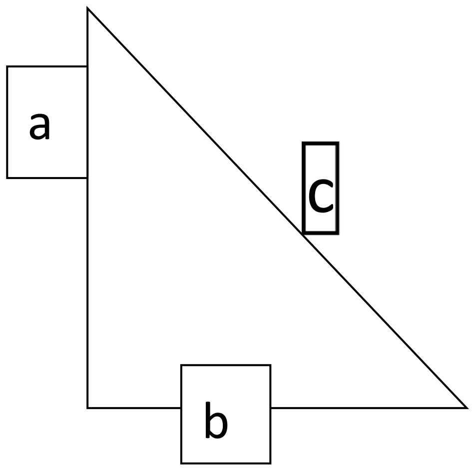
Trigonometry is the mathematical analysis of problem involving angles. Often using the trigonometric functions like sine, cosines and tangent. Within forensic science we may need to interpret data arising from measurements made at a crime scene. For example, in blood pattern analysis or bullet ricochet or to investigate cases of forensic relevance such as the trajectory of a rifle bullet or a suspicious death resulting from a fall from a tall building in all these investigation we need to understand the basic principle of trigonometry.

In this chapter we shall start by receiving the basics of trigonometry, particularly by measurements of angle and the definitions use and manipulation of the sine, cosines and tangent function

PYTHAGORAS THEOREM

'The square of the hypotenuse is equal to the sum of squares of other sides'

It only works for a right angled triangle. While using it is important to identify the hypotenuse correctly at the start. Pythagoras originally designed his equation by considering the relative areas of one square within a large square where the corner of the first square are just touching the sides of the second.

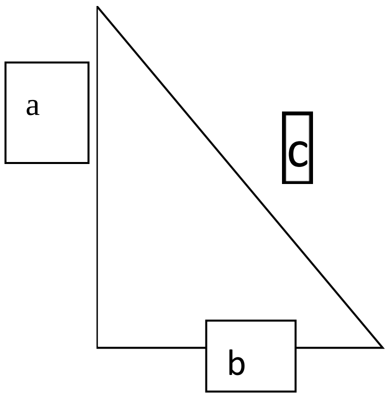


o $a^2+b^2 = \boldsymbol{c^2}$

THE TRIGNOMETRIC FUNCTIONS

Consider any triangle and imagine magnifying it so that it becomes larger. If the triangle is right angled then each angle is uniquely specified by the ratio of any two sides. This special links between an angle and the ratio of two sides for a right angled generates trigonometric functions of that angle which are of immense use in scientific calculations. There are three principle trigonometric functions which are called the sine, the cosine and the tangent of the specified angle θ . These functions are defined as

sin $\theta = \frac{a}{c}$



cos $\theta = \frac{b}{c}$

Tanθ = $\frac{a}{b}$

Using the trigonometric definitions we can deduce further useful equations

Tan$\theta = \frac{sin\Theta}{cos\Theta}$

θ	0^0	30^o	45^o	60^o	90^o
sin θ	0	0.5	0.70	0.96	1

cos θ	1	0.86	0.70	0.5	0
tan θ	0	0.57	1	1.73	∞

Above given table gives the summary of some trigonometric functions

<u>USE OF TRIGONOMETRY IN BLOOD SPLATTER</u>

Trigonometry holds a Special place in the hearts of the blood spatter analyst. The blood spatter analyst work very hard using the known identities and trigonometric functions to find the angle of impact, that is, at what angle was the person struck, and at what angle the blood fell. Trigonometry is also used in determining the height of a person, using the angle impact.

Blood Spatter:

When we speak of blood spatter, we mean the way that the blood is being distributed, the shape of the droplets when it hits the surface, and the angle of impact there are many types of blood spatter. Because blood doesn't always just drip out of the wound we don't always have perfect droplets of blood. Instead we get what looks like smears,, a droplet followed by an elongation (tear drop shaped) and or many tiny little droplets distributed about the surface they landed on.

The blood stain which is left is always bigger than the actual droplet; this is because the volume of blood is dispersed on the surface. When constructing a right triangle to determine at which angle the droplet fell, we must be certain that the angle outside the right triangle is equal to the right triangle on the inside. This way we can measure the bloodstain (as seen in figure 1).

According to Yonder, Anita, when a blood drop appears to be tear drop shaped, it is several cross sections of the sphere (droplet). Something to keep in mind when determining what type of spatter you are dealing with is that chords diameter is completely dependent on the velocity of the drop.

When a blood drop falls it will accelerate according to the gravitational force. It will then continue to fall until reaching equilibrium with gravity, and then come to a uniform velocity this is known as the Theory of Terminal Velocity. This theory was applied to blood spatter in order to gain some sort of reference sample of a constant velocity and known angles.

However, the problem in using this theory is that it depends on the mass of the blood drop. Blood is not uniform because people have different proportions of blood composition. Due to these different Proportions variance is expected within the individual because of the different ratios within the many different organs.

DIFFERENT TYPES OF BLOOD SPATTER

Low Velocity Spatter

Low velocity spatter is one we are familiar with; it is anywhere from 1.5m/s or less. This spatter is formed after we have “received” our wound.

For example, if I was to get cut in the arm and blood began to drip from the wound, I would then walk towards my first aid kit. The blood which hit the ground would have simply been dripping from my arm; there was no force applied to cause it to do anything else other than drip (see figure2).

The drops which landed on the ground would be droplets. When the drop is travelling slowly the diameter of the chord will be wider as seen in figure 3.What is interesting about blood is that it will increase in size the greater the distance is at which it falls, but will remain constant after 4ft.

When dealing with low velocity spatter, we are able to apply the theory of terminal velocity. Why? We are able to do this because terminal velocity does not deal with force; as you would have to take into account when dealing with fast velocity blood spatter. This type of blood spatter gives an almost perfect sphere, but there are a few tails called spines surrounding the bloodstain left behind. These stains are about 3mm or larger diameter.

Medium Velocity Spatter

Medium velocity spatters are produced with more energy and force than low velocity spatter. This is because medium velocity deals with force, whereas low velocity is dependent on gravitational force. Medium velocity spatter is given by blunt force and stabbings.

When droplets are dispersed they break off into smaller droplets of blood (see figure 4). When dealing with a stabbing, the bloodstain pattern will be relatively linear (see figure 5). This is because the surface area of the object is small and less blood being deposited from the wound. However, when dealing with blunt force, the blood spatter left will be varied in size because the surface area is larger.

High Velocity Spatter

Forensics.com). The spatter which is associated with high velocity is 2mm in diameter or less, and the force that produces this spatter is

100ft/s! High velocity spatter deals with gunshots, high speed collision, and explosions**.** The bloodstain pattern given off by high velocity spatter looks like a mist (see figure 6), Because of the high velocity.

Let us note that when we say velocity we mean the measure of force that has been applied to the blood. With some high velocity spatter some of the spatter may travel backward toward the gun, known as Back Spatter.

However, if the spatter moved in the direction that the bullet was travelling this would be known as Forward Spatter.

When the gun is closer to its target the spatter will be greater in dispersion, the same applies to the bullet, the larger it is the more dispersion there is. The spatter tends to be 2mm to 4mm in diameter, and the force which causes this type of spatter is 25ft/s (Crime scene).

ANGLE OF IMPACT:

The angle of impact tells us the angle at which the blood hit the surface. For instance, if my arm were bleeding and I held it straight out the angle would be 90 degrees.

<u>To calculate the angle of impact:</u>

We take the width measurement and the length measurement of the blood stain, keeping in mind not to measure the tail, (see figure 7). We do not measure the tail because it is

caused by gravitational force and the force of the weapon. We should expect to see a larger tail the smaller the angle of impact, see figure 8.
To calculate, do the following angle of impact tells us the angle at which the blood hit the surface. To calculate do the following

Ex 1: Let us say that the width is 9mm and the length is 18mm STEP 1: sin^{-1}(9mm/18mm) = 30˚

Ex. 2: Let us say the width is 1.5 cm and the length of the blood stain is 3.0cm
STEP 1: sin^{-1} (1.5cm/3.0cm) = 30˚
The angle of impact was 30˚.

EX 3: Let us say that the width is12mm and Length is 18mm

Sine of impact angle=width of bloodstain/length of bloodstain

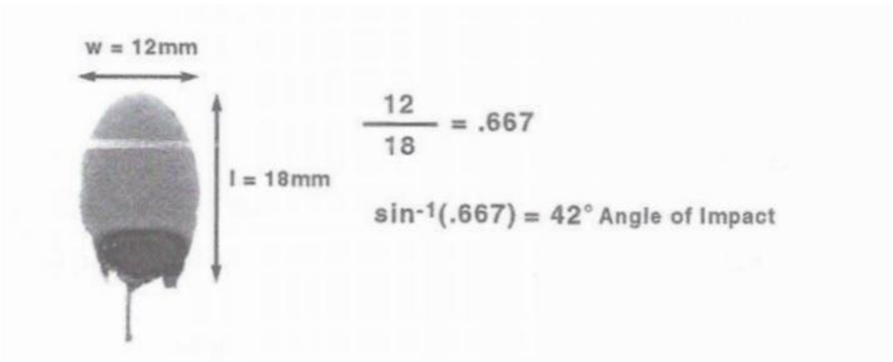


FIGURE A

AREA OF CONVERGENCE

The area of convergence tells us where the spatter may have originated. To do this strings are taken and are attached to each blood stain down its axis (see figure 9), this will show us where they

Converge (see figure 10) after stringing we are able to see if the spatter is moving in an upward direction or a downward direction.

Sometimes looking at the tail isn't enough because the gravitational force will simply pull it down anyway. When dealing with blunt force blood spatter, it is important to look up to the ceiling, as the blood spatter there is likely from the blood that was on the object and was put there due to the swinging of the object.

It is important to look at Directionality, because this will show you which spatter are due to swinging the object backward and which is due to forward motion. If we know where the perpetrator was, we can get a much clearer Understanding on whether the person is right handed or left handed.

There are times when the assailant will attempt to make a killing appear as a suicide. Area of convergence plays a very big role in this. It shows if there are any discrepancies. The beautiful thing about math is that it doesn't lie. If the area of convergence doesn't match where the body should be, this is a good time to start thinking that foul play may be involved. There are several times in which there is no blood spatter but a gun was used for the "suicide". Because a gun is high velocity it is likely that the spatter is small.

DETERMINING HEIGHT

It is important to determine the height at which the blood fell, because it tells us the height that the blood drop originated from. It is possible that the victim and the assailant both leave spatter evidence. Due to this, it is important to take notes of the heights and find where any inconsistency may lay. Knowing the height of the victim we may deduce that a certain blood stain originated from the Perpetrator or the victim.

EX 1: Let us say that we find a blood stain 10ft. away from the source and we have determined that the angle of impact is 70˚. To determine the height we construct the following.

Now we calculate:

STEP 1: tan 70°= X/10

STEP 2: 10(tan 70°) = 27.5ft.

The height at which the blood fell was 27.5 ft.

Ex 2: Let's say that the blood stain was 15ft. away and the angle of impact was 80°

STEP 1: tan 80°= X/15

STEP 2: 15(tan 80°) = 85.1ft.

The height at which the blood fell was 85.1ft.

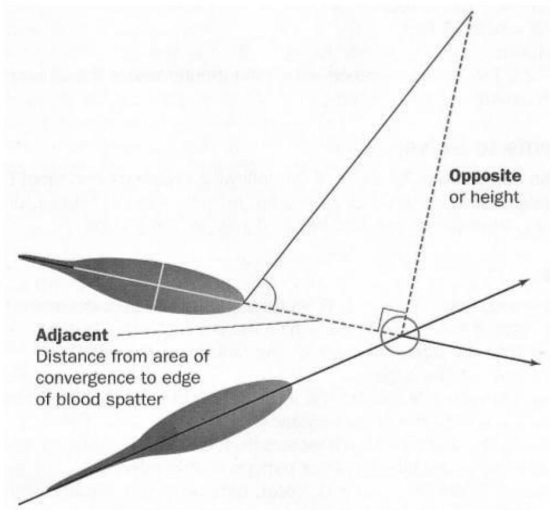


FIGURE B

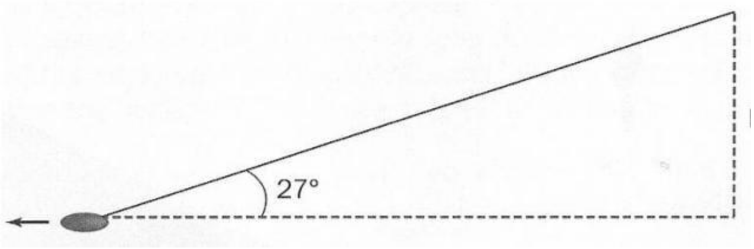
Tangent of angle of impact = opposite / adjacent

= height / distance

Height = tangent of angle x distance

Example:

Crime scene investigators noted blood spatter on the floor of the kitchen. The investigators drew lines of convergence and measured the distance from the area of convergence to the front edge of a drop of blood. That distance was recorded as 5.75 feet. After measuring the length and width of the blood droplet and using the law of sines, it was determined that the angle of impact was 27 degrees. The police wanted to determine the point of origin, or the height from the floor where the person was bleeding.



Height

Distance to area of convergence = 5.75 ft

FIGURE C

Solution:

Tan = opposite / adjacent

=height / distance

Or

Tangent of blood-spatter angle = height of wound / distance from blood to area of convergence

tan 27^o = height of wound / distance

= height / 5.75 ft

Height = tan 27^{o} x 5.75 ft

Height = 2.9 ft

CONCLUSION

As you have seen, Trigonometry has a large and important role in bloodstain analysis. Without it we would not be able to find the height, angle of impact or area of convergence. Unfortunately, there are murders, and unfortunately for the murderers trying to flee there is trigonometry. Where there is blood stain, there is trigonometry and “absence of evidence is not evidence of absence.”

FIGURE 1

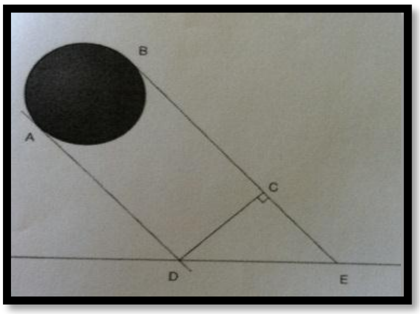


FIGURE 2

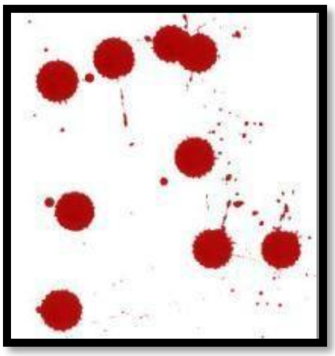


FIGURE 3

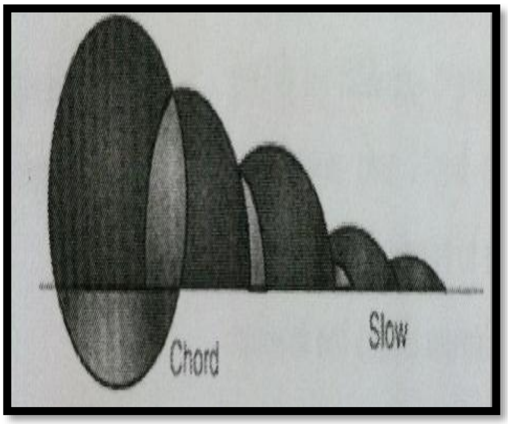


FIGURE 4

FIGURE 5

FIGURE 6

FIGURE 7

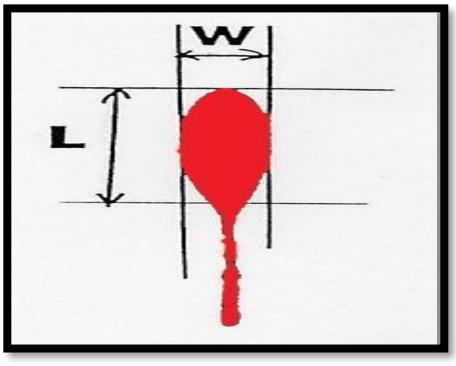


FIGURE 8

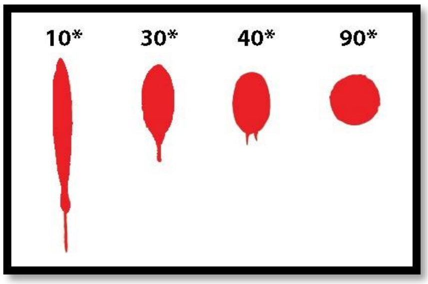


FIGURE 9

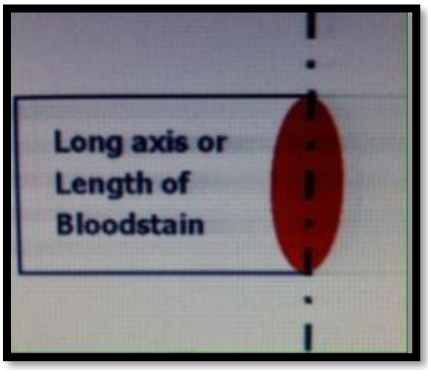
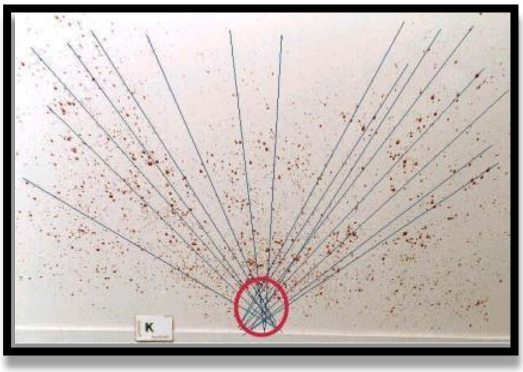


FIGURE 10



APPLICATION OF HEIGHTS AND DISTANCES IN CRIMINAL INVESTIGATION

Here we are investigating how direct application of Trigonometry may be used to solve some simple problem in forensic science .In determining heights and distances from a set of measurements the basic principle is that each example may be reduced to triangle or group of triangle and the Trigonometric functions and rules are then used to calculate any unknown length or angles

Example:

A burglar gain access to an up stair window by using a ladder. If the height of the window is 4.3m above the ground and impression mark from the ladder are found 1.2m out from the wall .Calculate the length of the ladder and the angle it makes to the ground?

The ladder forms the hypotenuse of a right angled triangle with the ground and the wall. Let the length of the ladder be L, we use Pythagoras theorem to calculate L

$L^2 = 4.3^2 + 1.2^2 = 19.93$

$L = \sqrt{19.93} = 4.46m$

Since the angle in question is opposite the wall, the sine of this angle θ is given by

$\sin\theta = \frac{4.3}{4.46} = 0.964$

$\theta = \sin^{-1} 0.964 = 74.6$

RICOCHET ANALYSIS

Ricochet occurs when a bullet bounds off a solid surface at a glance after impact and then continuous its trajectory. Unless the incident angle is below the critical angle for that surface , the bullet will either fragment on impact or penetrate the surface rather than ricochet .

In almost all cases the ricochet angle or at which the bullet leaves the surface is lower than the incident angle tan θ_i and these parameters are linked approximately by the equations

$\frac{\tan \Theta_r}{\tan \Theta_i}$ = c, where c is a constant

CHAPTER 3

THE DETERMINATION OF THE ESTIMATED TIME OF DEATH USING DIFFERENTIAL EQUATIONS

Differential equation deals with the derivatives. It is an aspect of pure mathematics that provides solutions to many physical problems

An equation that involves one or more derivatives of unknown function is called differential equation. The order of highest derivative on a differential equation is referred to as the order of the equation. Differential equation arises in a derivative that is whenever a certain quantity changes with respect to another. If unknown is a function of several variable the equations is referred to as partial differential equations

The main objectives of study are

1. To show how differential equations can be utilized to solve some physical problems
2. To investigate the application of differential equation in mathematical modelling
3. To translate some physical situation in to mathematical problem and to solve the resulting problem.

THE TIME OF DEATH

There are several times of death. It seems to be simple and straight forward term that obviously means the exact time that the victim draws his last breath. There are actually three different times of death

1. Physiological time of death, where the victim's vital functions actually cease.
2. The legal time of death, the time recorded in the death certificate.
3. The estimated time of death, the time medical examiner estimate that death has occurred

If you are asked to tell the estimated time of death. We use the concept of Newton's law of cooling

Newton's law of cooling is differential equations that takes two inputs first the temperature at a given time and second the temperature at any arbitrary time. Because this equation can have infinitely many outcomes depending on the inputs, there is a constant which is associated with it. This constant can only be found from the two inputs and the equation can only be solved with the constant.

Newton's law of cooling relates the temperature of an ordinary body and the temperature of the body's surrounding medium, which is the source of the heat induction or heat loss of the body, and the change in temperature. The constant is the unique factor that depends on a particular solution.

Newton's law of cooling

The rate at which the temperature T(t) changes in a cooling body is directly proportional to the temperature difference between the body and the temperature T_S of the surrounding medium.

Newton's law of cooling modelled as first order initial value problem

$\frac{dT}{dt}$ = K (T-Ts)

T(0) = T_0

T_0 is the initial temperature of the body and K is the constant of proportionality If T_s is constant by method of separation of variables

$\frac{1}{(T-T_S)}$ dt =

K dt

Integrating both sides

$\int\frac{1}{T-T_S} = \int K\,dt$

log($T - T_s$) = Kt + c

(T-T_s) = e^{Kt+c}

T (t) = T_s +ce^{Kt}

Applying the initial condition

T (0) = T_s +ce^0

C=T_0 -T_S

Hence T (t) = T_s + (T_0 - T_s) e^{Kt}

If K<0 $\lim_{t\to\infty} e^{Kt}$ = 0

Therefore, $\lim_{t\to\infty} T(t)$ =T_s and the temperature of the body approaches that of its surroundings

POSTMORTEM COOLING

During life, body temperature is maintained by the body's metabolism. At death metabolism ceases and body temperature begins to approach the ambient temperature which is always lower than body temperature. Body warmth is lost from the surface to the surroundings, the core temperature remain relatively static for the first one to two hours after death.

Normal body core temperature is 98.6 F and certainly many individuals die with that particular body temperature. However there are factors may raise or lower a person's body temperature at death.

According to splitzetal body cooling is fairly complex and relays on a variety of body mechanisms. The factors that affect body temperature after death are humidity, insulation, surface in contact with the body, environment temperature.

Careful consideration of all factors will not allow a pinpoint documentation of time of death but at least a range for time of death can be found. The range of death is the period of time in which the death is believed to have occurred.

When estimating the time of death one need to know the temperature of the surroundings and the body temperature in order to make an accurate estimate

EXAMPLE 1

At 9am on October 19, 2009, a body was found in room 327 at the university centre. The room is kept at a constant temperature of 72F. The medical examiner was called and he arrived in 8 minutes. The first thing he did was to take the temperature of the body, it was 83F. Thirty minutes later the temperature of the body was taken again and it was now 78F. Help the police by telling them when the person was murdered?

The initial condition is taken as

T_0=83 and T_s =72 at 9.08 am

Substituting these values in

T(t) = T_s + (T_0 - T_s) e^{Kt} = 72 +(83-72) e^{Kt}

=72 +11$e^{Kt} \rightarrow eq$ 1

At 9.38 when the medical examiner took the body temperature T(30)=78 and T_s =72 again substitute as above we get

78= 72+ 11e^{Kt}

78- 72=11e^{Kt}

6=11 e^{Kt}

$\frac{6}{11}$ = e^{Kt}

log $\frac{6}{11}$ = Kt

Thus we get K=-0.0202 where t= 30

Substituting K in eq1

T(t)= 72+ $11e^{-0.0202t}$ for t

This equation takes the form of exponential decay due to its negative exponent.

Since the medical examiner wishes to know the time of death a solution for t is found by setting the temperature of body at 98.6F, normal body temperature

Now solving we get

98.6= 72+ $11e^{-0.0202t}$ for t

98.6 – 72= $11e^{-0.0202t}$

$\frac{26.6}{11} = e^{-0.0202t}$

$\log \frac{11}{26.6}$ = 0.0202t

t = $\frac{\log\frac{11}{26.6}}{0.0202}$

= - 43.7

Subtracting 43 minutes from 9.08 am gives the time of death.

That is 8.25 am

EXAMPLE 2

Suppose that a corpse is found in their room in a hotel at 10pm, its temperature is 30 degrees Celsius. The room is kept constant at 25 degrees Celsius. 2 hours later the corpse’s temperature drops to 26 degrees Celsius. Find the time of death?

1. Find the constant k

$$k = \frac{-1}{2}\ln\left(\frac{26-25}{30-25}\right) = 0.8047$$

2. Assuming the temperature of the corpse at the time of death was 37 degrees Celsius, we solve for the time of death by just rearranging the equation

$$t = \frac{-1}{k}\ln\left(\frac{T(0)-Ts}{T(t)-Ts}\right)$$

$$t = \frac{-1}{0.8047}\ln\left(\frac{37-25}{30-25}\right) = -1.0879\ hours$$

3. Minus this from the time the body was found and then convert the decimal to hours

10-1.0879=8.9121, time of death was 8:55pm

EXAMPLE 3

At 10:07 pm, you find a secret agent murdered. Next to him is a martini that got shaken before the secret agent could stir it. Room temperature is $70^0 F$. The martini warms from 60^0F to 61^0F in the 2 minutes from 10:07pm to 10:09pm. If the secret agent's martinis are always served at 40^0F. what was the time of death?

$\frac{dT}{dt}$=K(T-T_s)

$\frac{dT}{dt}$=K(T-70)

$\frac{dT}{K(T-70)}$=dt

$\int\frac{dT}{K(T-70)}$=∫dt

$\frac{1}{K}\int\frac{dT}{T-70}$=t+c

$\log|T-70|$=Kt+c

|T-70|=$\pm e^{Kt}e^{c}$

T-70=-10e^{Kt}→eq1

61-70=-10e^{2K}

K= $\frac{\log\frac{9}{10}}{2}$

Eq 1→T-70= -10 $e^{\log\frac{9}{10}\frac{t}{2}}$

40-70= -10 $e^{\log\frac{9}{10}\frac{t}{2}}$

3= $(\frac{9}{10})^{\frac{t}{2}}$

t = -21 mint

Time of death = 10:07-00:21

= 09.46pm

CHAPTER 4

APPLICATION OF GRAPH THEORY IN CRIMINAL INVESTIGATION

INTRODUCTION TO GRAPH THEORY

Graph theory is a device used to inspect the relationship between the different entities. A graph G (V, E) is a collection of vertex set V(G) and edge set E(G).

DEFNITIONS

- Assignment of colours to vertices or edges of the graph with a label called graph colouring.
- Assignment of colours to the vertices of the graph such that adjacent vertices are assigned distinct colours is called as vertex colouring.
- Assignment of colours to the edges of the graph such that adjacent vertices are assigned distinct colours is called as edge colouring.

- If any of the k-colours are assigned to the vertices of the graph then graph is said to be k-colourable.

- If the graph is k-colourable then k colours are required to colour the vertices of such that no two adjacent vertices share the common colour which is called the chromatic number of the graph.

Graph theory is used to inspect the unstructured data in social network where the datasets are represented as vertices and their relationship is represented as edges. In the investigation of crime, graph theory is used in forensic examination to identify the suspects where the suspects are represented as vertices and their acquaintances are represented as edges.

Next, we are going to describe the methodology to identify the suspect by examining the entire crime through graph theoretic approach which acts a model to investigation of offence.

APPLICATION OF GRAPH COLOURING TECHNIQUES TO INVESTIGATE THE OFFENCE

A. GRAPH THEORY IN THE SCENE OF CRIME

In crime science, mathematical model is used for investigation. With the advent of computers, the modelling techniques represent the aspect of criminal behaviour and the analysis results in numerical solutions. These models helps the authorities to expedite the investigation process aimed at reducing time in decision making or to develop various strategies.

In this paper, the method deals with the evidence in question and estimating the connections between individual evidence suspects, victims, and other entities connected with investigation. This method analyses three elements of the graph such as the vertex set, the edge set, and the incidence function that relates edges to vertices. The mathematics of graph theory explains that an edge can be incident from one vertex to another if there is any connection between the entities.

The first step in applying graph theory to any investigation is to identify the various elements involved in the incident. These elements will be represented as vertices. When any two elements have any connection that connection is represented as an edge. The edge not only connects the vertices but acts as the initiator of that connection which results in graph with parallel edges.

For example, if the investigation includes confidential data that was robbed from firm $F1$ and subsequently found in firm $F2$ then consider each firm as a vertex and the connections between these employees from firm $F1$ to firm $F2$ would be represented as edges. The key acts as a modelling tool to ensure that all entities and connections are represented in a graph for various classes of crime

B. METHODOLOGY TO CONSTRUCT GRAPH FOR INVESTIGATION

To illustrate the methodology of graph colouring technique in investigations of crime here we provide the algorithm which clearly depicts the formation of graph.

Consider the scenario involving a crime where a jewellery shop has been subjected to a robbery. Let us assume the suspects U, V and W as vertices. Assign the colours blue, red and green to the vertices. Suspect U says he is not the robber then lines to be drawn from U to V and W and assign colour blue. Suspect V says U is the robber then there is a line connecting from V to U and assign colour red. Suspect W says he is not the robber then lines to be drawn from W to U and V and assign colour. This represents the graph with three vertices and connected by edges to identify the robber.

Algorithm for Construction of Graph G

Step 1: Consider the suspects as vertices U,V and W.
Step 2: Assign distinct colour to the vertices blue, red and green to U, V and W. Step 3: Represent the connection between the vertices U, V and W as edges and assign the respective colour to the edges.
Step 4: Suspect U says he is not the robber then draw lines from U to V and W.
Now assign the colour blue to the edges.
Step 5: Suspect V says U is the robber then draws lines connecting from V to U.
Now assign colour red to the edges.
Step 6: Suspect W says he is not the robber then draw lines from W to U and V. Now assign colour green to the edges

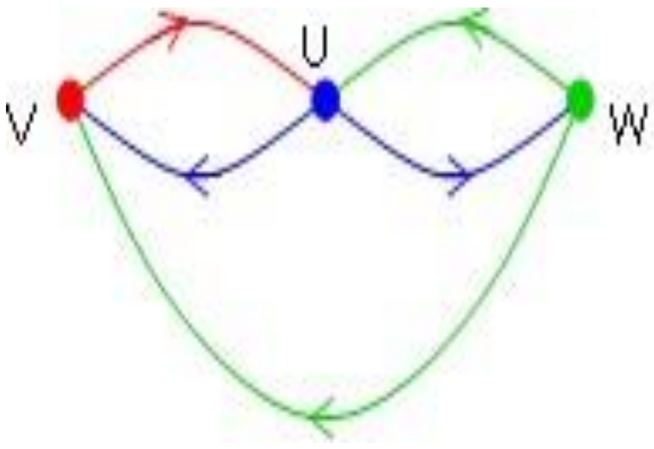


Fig. 1. Crime graph G for three suspects

III. RESULT AND DISCUSSION

C. DISCUSSION OF CRIME PROBLEM AND ROLE OF COLOURING TECHNIQUE IN CRIME INVESTIGATION

Proposition 1:

In the scene of crime, police arrested three suspects for investigation. Suspect U says he is not robber. Suspect V says U is the robber. Suspect W says he is not the robber where one of the suspects says truth out of the three. The information of the suspects is given in the following table. Find out the robber in this crime scene?

SUSPECTS	INVESTIGATION
U	**I am not the robber**
V	**U is the robber**
W	**I am not the robber**

PROOF:

Consider the crime graph G where the vertices U, V, W are suspects and connection between is represented as edges. Assign the colours blue, red and green to the vertices. Suspect U says he is not robber then lines to be drawn from U to V and W and assign colour blue. Suspect V says U is the robber then there is a line connecting from V to U and assign colour red. Suspect W says he is not the robber then lines to be drawn from W to U and V and assign colour green. This represents the graph with three vertices and connected by edges to identify the robber.

Case 1: Police suspects U to be the robber

Suppose if U is the robber then V or W are not the robbers. Here V is telling truth as he says U is the robber. Also V and W become liars. If so W says he is not the robber which is a contradiction which violates one person says truth. Hence U is not the robber.

Implementation of colouring technique for case 1:

Suppose if U is the robber then redraw the graph G by removing all the lines coming out from U. Here V is telling truth as he says U is the robber so there is a line from V to U. Here W says he is not the robber so remove the lines coming out from W which is a contradiction to assumption which is clearly depicted in graph G1. Graphically, two lines pointing towards A which is a contradiction to one person says truth. Hence U is not the robber.

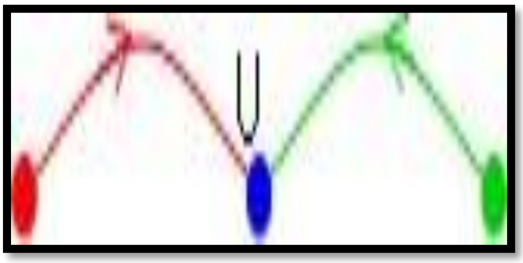


Fig. 2. 1st level investigation graph G1

Case 2: Police suspects *V* to be the robber

Suppose if V is the robber then U or W are not the robbers. Here U is telling truth as he says he is not the robber. Also V and W become liars. Here V becomes liar as he says U is the robber. But W says he is not the robber which is a contradiction which violates one person says truth. Hence V is not the robber.

Implementation of colouring technique for case 2:

Suppose if V is the robber then redraw the graph G by removing all the lines coming out from V. Here U is telling truth as he says he is not the robber so remove the line from W to U. But W says he is not the robber which is a contradiction which violates one person says truth which is clearly explained in the graph G2. Also graphically, two lines pointing towards V which is a contradiction. Hence V is not the robber.

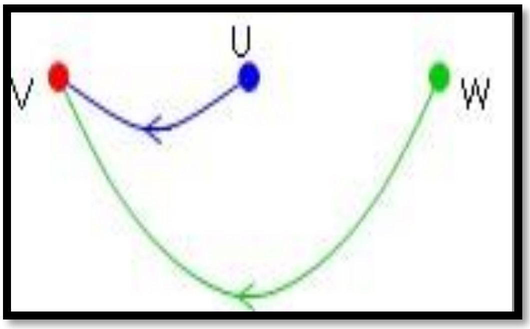


Fig. 3. 2nd level investigation graph G2

Case 3: Police suspects *W* to be the robber

Suppose if W is the robber then V or U are not the robbers. Here U is telling truth as he says he is not the robber. Clearly V become liar from his statement. Also W become liar as V says U is the robber. This case is logically consistent where one person telling truth. Hence W is the robber.

Implementation of colouring technique for case 3:

Suppose if W is the robber then redraw the graph by removing all the lines coming out from W. Here U is telling truth as he says he is not the robber so remove all lines pointing towards U. Also, V says U is the thief who is a liar hence remove the line from V to U which is clearly demonstrated in the graph G3. Graphically there is a single line from W to U which is logically consistent where one person telling truth. Hence W is the robber.

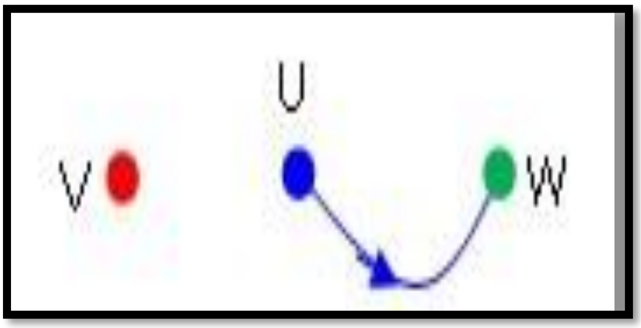


Fig. 4. 3rd level investigation graph

Thus, we arrive at the solution for the crime problem using the graph colouring techniques during the investigation process which expedites the crime scientist to reduce time factor for detecting crime.

EXTENSION OF PROPOSITION 1

Here we discuss about the case of more suspects for a complicated set of acquisitions. For example, we extend the robbery case to four suspects in the crime scene . the information of the suspects are given in the following table. Find out the robber involved in this crime?

SUSPECTS	**INVESTIGATION**
P	**I am not the robber**
Q	**P is the robber**
R	**I am not the robber**
S	**Q is the robber**

Proof:

Consider the crime graph G* where the vertices P, Q, R, S are suspects and connection between is represented as edges. Assign the colours blue, green, red and yellow to the vertices. Suspect P says he is not robber then lines to be drawn from P to Q, R and S and assign colour blue to the corresponding vertex and edges. Suspect Q says P is the robber then there is a line connecting from Q to P and assign colour green. Suspect R says he is not the robber then lines to be drawn from R to P, Q and S and also assign colour green. suspect S says Q is the robber then there is a line connecting from S to Q and assign colour yellow. Hence this graph clearly reveals the connection between the acquisitions. This represents the graph with four vertices and connected by edges to identify the robber.

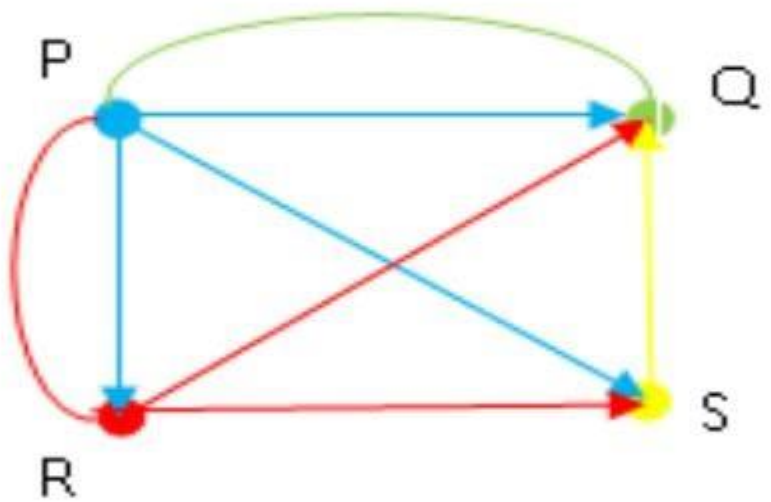


FIGURE 5. Crime graph G* for four suspects

The proof is obtained by proceeding in the same method as proposition 1. Here we arrive at the following sequence (2,3,1,2) is the solution set for the result of investigation of

crime problem which indicates the number of lines coming towards (P, Q, R, S). This sequence clearly detects the number of person telling truth if the person is the robber.

we can find possible solutions directly by counting number of lines coming in from the vertices which detects the number of people telling truth.

Observations:

1. In this extension of proposition 1, we arrive at the following sequence (2,3,1,2) is the solution set for the result of investigation of crime problem which indicates the number of lines coming towards (P, Q, R, S). This sequence clearly detects the number of person telling truth if the person is the robber.

2. In this extension of proposition 1, from the solution set it is clear that R is the robber for one person telling truth which is proved above in the cases of proposition 1.

3. For two person telling truth we arrive at choices as P or S. In this case, we cannot find the exact person as it leads to choice instead we can say definitely not Q and R.
For three person telling truth we arrive at accurate solution that Q is the robber. These solutions helps the crime scientist to identify the criminal for lower number of suspects directly.

CHAPTER 5

PROBABILITY IN FORENSIC SCIENCE

The term probability is given to a formal measure of the certainty that a particular event or outcome will occur. In each cases of probability the result is based on a unbiased outcome where every possible result is equally likely.

In forensic science empirical probabilities are particularly important and sample may be derived from the data on height, finger print class, blood group, allele frequencies in DNA.

CALCULATING PROBABIILITY;

The fundamental assumption of many probability calculation is that all outcomes are equally likely and either head or tail up. But which of these outcomes will occur on given occasion cannot be predicted

Probability = $\frac{number\ of\ selected\ outcomes}{total\ number\ of\ possible\ outcomes}$

APPLICATION: PROBABILITY AND HUMAN TEATH MARKS

The matching of bite marks, for example from indentations on the skin of a victim, against a

reference set of marks from a suspect may be based on quantitative measurements of the position and orientation of each tooth with each set. For each tooth there are six classes of position, termed the bucal, lingual, mesial, distal, mesial rotation and distal rotation. However, detailed examination and measurement of 384 perfect sets of test bites has revealed that greater resolution was possible in terms of determining the position within the mark and the orientation angle of each tooth . These parameters were shown to follow frequency distributions with fairly steep cut- off at the edges. By assessing the errors in such measurements as ±1 mm in determining the centre of the tooth and $\pm 5^{o}$ in each angular orientation, these distributions suggested that between 100 and 200 distinct positions were available for each of the 12 teeth involved in this study. The average number of distinct positions was approximately 150. The total number of distinct positions over this set of 12 teeth is therefore given by 150^{12}.

If it is assumed that the observed frequency distributions give some support to the proposition that each tooth position is equally likely, then the probability of any single distinct set of 12 tooth marks is given by:

$$P = \frac{1}{150^{12}} = 1.3 \times 10^{-26}$$

As the global population is around 6700 million, such a probability justifies the conclusion that a well resolved set of bite marks may be individualized through this methodology.

APPLICATION : THE MATCHING OF HAIR EVIDENCE

The macroscopic examination of hair evidence involves the individual assessment of many features of each hair such as colour pigment, density, medulla characteristics and physical dimension. The forensic scientist will normally carry this as a comparison process between the pair of hair.

The work by GAUDETTE AND KEEPING has shown that for an experience scientist working on paired Caucasian, scalp hairs from different individual, nine pair were indistinguishable of all 366630 paired hair comparisons. This gives the probability that any two hairs taken at random from each of two people will be indistinguishable as

$$P = \frac{9}{366630} = 2.45 \times 10^{-5}$$

This implies that the probability of distinguishing this hair is 1-P. It is common practise to compare a questioned hair with a set of n randomly chosen dissimilar reference hair from the same head using the first rule of combining probabilities. The probability that all n will be distinguished from the questioned hair is given by

$P_n = (1 - P)^n$ =1-nP

The approximation is valued when P≤ 1 and is called the binomial expansion approximation. This probability applies to a result where hairs from different individuals are correctly discriminated. Thus the complimentary probability that the questioned hair is indistinguishable from at least one of the n reference hair is simply nP.

PROBABILITY IN SOLVING CRIMES

According to Kim Rossmo, his research resulted in a new criminal investigative methodology called geographic profiling, based on Rossmo's formula.

His formula aims to find the location of where the offender might be living. Based on the studies, it is seen that criminals commit their crimes in an area not too close but also not too far away from where they live.

We can find an area of possible locations:

In the diagram, the green spots highlight where the crimes have been done. The area called “HOT ZONE” is the area with the highest probability where the criminal resides.

He uses the following formula:

$$p_{i,j} = k \sum_{n=1}^{\text{(total crimes)}} \left[\underbrace{\frac{\phi_{ij}}{(|X_i - x_n| + |Y_j - y_n|)^f}}_{1^{\text{st}} \text{ term}} + \underbrace{\frac{(1-\phi_{ij})(B^{g-f})}{(2B - |X_i - x_n| - |Y_j - y_n|)^g}}_{2^{\text{nd}} \text{ term}} \right]$$

We divide the area into grids and the equation shows the probability in a particular grid.

The first term essentially the denominator, measures the distance from the point of crime till the grid.

HIGHER THE DISTANCE → HIGHER THE DENOMINATOR → HIGHER THE PROBABILITY OF THE GRID BEING THE RESIDENCE AREA OF THE CRIMINAL.

The second term essentially the denominator, subtracts the distance from a buffer zone (B). All of the other variables in the equation remain constant.
HIGHER THE DISTANCE → LOWER THE DENOMINATOR → SMALLER THE PROBABILITY OF THE GRID BEING THE RESIDENCE AREA OF THE CRIMINAL.

Both these terms help balance the distance from being too close or too far from the crime scene. When we add these, we can make the HOT ZONE.

CONCLUSION

The main use of forensic science is for the purpose of law enforcement to investigate crime such as murders, theft. The techniques developed by forensic science are also used by the army to analyse weapons, high explosives etc...

The importance of maths in the administration of justice has risen with the growth of identification forensics and its influence continues to permeate questions of proof and judgement. For example statistics (evidence).

Mathematics is fast becoming one of the most important techniques in crime detection. Where once a Sherlock Holmes would have to be

Content with a magnifying glass or a jury with gut instinct and rational discussions now a range of methods from probability and statistics are available to help. Today mathematics leaves behind expert conclusion on a hundred forensic matters from finger prints to DNA.

Scientific techniques and old techniques were sufficient in solving the crime problem. But as the society grew major developments took place the accused person become more and more aware of the use of scientific techniques in committing the crime due to ineffectiveness of the old techniques. Here comes the importance of mathematical techniques in criminal investigation.

REFERENCES

- Essential mathematics and statistics for forensic science; CRIAG ADAM
- The determination of the estimated time of death using differential equation; FAITHU MUHAMMED AND AUWALU HARUNA