MODELLING OF THE RADIAL VELOCITY SIGNAL FROM STARS WITH EXOPLANETS

A project submitted to MG University in fulfillment of the requirements for the award of the degree

MSc. Space Science

Submitted by

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DEPARTMENT OF EARTH AND SPACE SCIENCES

INDIAN INSTITUTE OF SPACE SCIENCE AND TECHNOLOGY

CERTIFICATE

This is to certify that the project report entitled "Modelling of the Radial Velocity Signal from Stars with Exoplanets" is an authentic record of the project work carried out by **Ms.PARVATHY THANKACHY P (Reg.no: 200011024030)** in partial fulfillment of the award of the degree of Master of Science in Space Science at Bharata Mata College, Thrikkakara affiliated to Mahatma Gandhi University, Kottayam under my guidance and supervision.

PROJECT GUIDE

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DECLARATION

I, **PARVATHY THANKACHY P** do hereby declare that the rpoject report entitled "MODELLING OF THE RADIAL VELOCITY SIGNAL FROM STARS WITH EXOPLANETS" is a bona-fide record of the work submitted to Mahatma Gandhi University in the partial fulfillment of the requirement for the award of the degree of MSc in Space Science carried out by me under the guidance of **Dr. ANAND NARAYANAN**,Professor, Indian Institute of Space Science and Technology, Thiruvananthapuram.

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ABSTRACT

The interest of knowing if life exists in places other than Earth has existed since a very long time. With the finding and confirmation of the 51 Pegasi b, a Jupiter sized planet around a G-type star called 51 Pegasi paved a strong path to the search for exoplanets and to a hope of possibly find life in one among them. This first detection and confirmation was done by a technique called as the Radial Velocity method, a ground based technique that mostly uses the variations in the visible and near-IR spectrum to determine the presence of an exoplanet, has prevailed as one of the most important techniques to find exoplanets till this day. Studying this method is a first step towards knowing exoplanets, and their properties such as minimum mass, eccentricity, orbital separation, time period, etc. From the shifts observed, radial velocity data can be taken, which upon plotting can give details about these mentioned parameters. Modelling a radial velocity curve using these parameters not only let someone who only has an idea about these parameters visualize the motion but also allows one to know how the radial velocity of the planet changes.

Chapter 1

Introduction

The night sky has always astounded the mankind since the beginning. From just watching the sky like a beautiful piece of art, later on believing the stars to be just holes on a dome that covers earth and the starlight to be the light passing from the heaven above, to Earth through these holes, to the understanding that this very Earth on which we humans live is nothing but a small rocky planet revolving around a medium star Sun, and that there could be a million other stars with many planets around them, the advancements brought in by the element of curiosity in humans are not small. For thousands of years, humans have asked themselves many questions regarding the life on Earth and is it special. "Are we the only life forms existing in this universe?", "Is there another Earth with life like ours around any one of the stars in this universe?", " If ours is the only planet with life, doesn't that make us unique and special?", and many more questions exists and every human being must have asked one of these questions themselves at some point in their life.

A path to answering these questions paved its way when scientists detected signals which indicated that there could be a planet revolving a star and the possibilities became plausible with the confirmation of one such planet in 1992. Many planets around other stars have been discovered since then and many have been found to be in the habitable zone , indicating the possibility of finding life in one among them.As naked eye observation of these planets are not possible, several methods have been developed to detect exoplanets.

1.1   Detection Techniques

Upon looking up at the night sky, one can see thousands of stars as twinkling tiny dots. However, many of these stars are even bigger than the Sun. But they are very far away and so their angular diameter is very small, making them appear to be dots when viewed from Earth. The nearest star to Sun, Proxima Centauri, a low-mass star, is itself around 4.2 light-years away from the Sun. In comparison to stars, the planets that revolve around them are very small, making their angular diameter even smaller. In addition, the stars are a million times brighter than the planets that orbit them, making them further difficult to detect due to the glare from the stars.

Several techniques have been used to detect exoplanets such as Radial velocity technique, transit method, gravitational microlensing, direct imaging, astrometry, pulsar
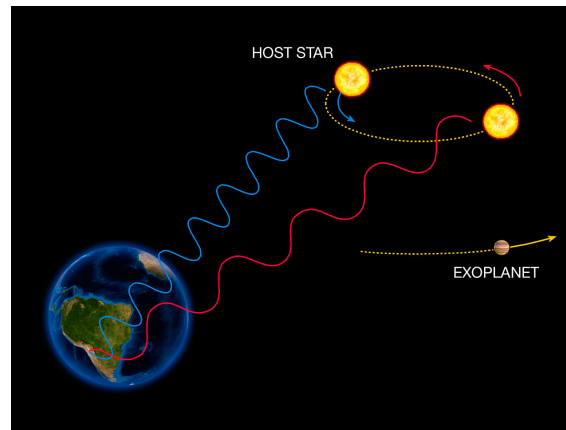
Figure 1.1: Doppler shift observed in a star due to its barycentric motion as a result of the presence of a planet. *Source:*[48]

timing method, etc. and these methods not just help in detecting exoplanets, but some even provide additional information regarding the planet. For instance, radial velocity technique helps to find the minimum mass the planet can have and transit method gives information about the size of the planet. While the said two methods focuses on planets that are in orbits closer to the host star, methods such as gravitational microlensing, direct imaging and astrometry focuses on planets that travel along wider orbits around the host star.

1.1.1 Radial Velocity Method

The radial velocity method is one of the most prolific techniques that has been used to detect exoplanets. This method measures the Doppler shift of the star as it moves around the barycenter (Fig: 1.1). This method not just helps in detection but also helps in finding the minimum mass that the planet can have. If this is combined with the transit method, true mass of the planet can also be calculated. The first confirmed exoplanet was detected using radial velocity method. This technique has helped in the detection of more than 900 exoplanets and has led to many milestone discoveries. Many hot Jupiters, multi-planet systems, planets around medium sized stars, planets around M-dwarfs,transiting planets[28], super Earths and many more [22][18].

1.1.2 Transit Method

Being the method with the maximum number of planet detections, the transit method or the transit photometry is one of the widely used techniques to study exoplanets. When a planet transits a star, there will be a decrease in the brightness of the star. Then, by studying this decrease in brightness for a period of time, planets can be detected.It is also possible to understand the size of the planet by using this technique. Consider a star-planet system in which the planet has a small size. As the planet transits, there will be a decrease in the brightness of the star. Now, consider the same star-planet system, but with a difference that the planet has a bigger size. Upon transiting, there will be a decrease in the brightness of the star but, upon comparing with the former case, it can be seen that the decrease of brightness in the latter is more significant than the former. So, larger the size of the planet, larger will the dip in the brightness of a given star. Thus, by analysing
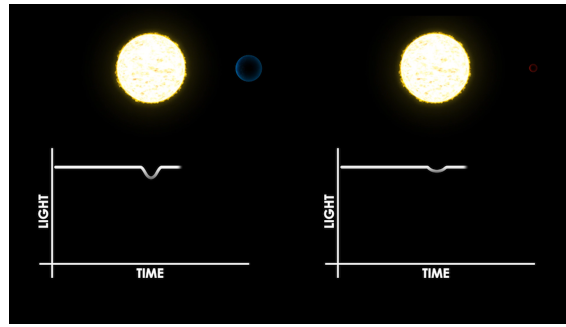
Figure 1.2: When a planet of bigger size transits a star, the dip in the light curve is more in comparison to the dip observed when a smaller sized planet transits. *Source:* [49]

the decrease of brightness in the light curve , size of the planet can be understood (Fig: 1.2). It also helps in understanding the atmospheric composition of the planet, leading to gain an understanding regarding its habitability. Upon combining with radial velocity method, more features of the planet could be studied, which both the techniques fail to study independently[13][18].

1.1.3 Gravitational Microlensing

Gravitational microlensing is a unique technique in which the gravitational perturbation of the light from a distant star is used to detect the planets belonging to another star. Suppose that there are two stars in the sky, a source star 'S' and a lensing star 'L'. These two stars are aligned in such a way that, when these are observed from a particular point O, star S falls right behind star L. In other words, L and S are perfectly aligned along the line of sight of the observer, who is at the point O. In such cases, the observer will not be seeing the source star, instead, will see an image I at a particular angle θ_E , above S and S will be lensed into an Einstein ring [18]. If the star L has a planet revolving around it, there will be a perturbation to the microlensing light curve of S, from which detection can be carried out. This method was developed in the late 1980s and has played a role in the detection of many massive planets that are bound to M-dwarfs, rogue planets and multi-planet systems. In addition, this technique is highly sensitive in detecting low mass planets that have very wide orbits [20][19]. Fig1.3 shows how lensing occurs.

1.1.4 Direct Imaging Technique

Direct imaging technique is the only direct method to detect an exoplanet. By taking a time series of images, orbital period of the planet can be calculated. Other properties of planets such as size, temperature, rotation rate, surface properties, etc.can be studied from the photometry,spectra and colours, making this method one of the powerful techniques available to study an exoplanet. This technique mainly uses infrared and also visible region to study its spectra. This technique also complements well with other exoplanet detection techniques. Having said that, direct imaging of a planet is affected by several factors. The greatest challenge faced by this technique is the brightness of the star in comparison to the planet. If the star is very bright with respect to the planet, planet will appear to be much fainter and while taking the image of the planet, there will be a presence of the residual light that is scattered by the planet from the incident light of the star.This
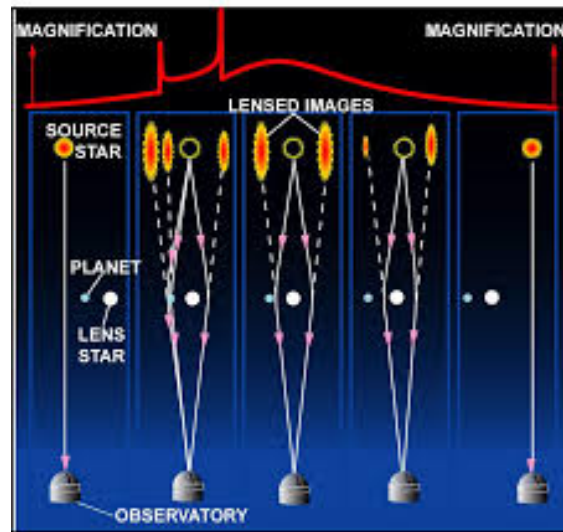
Figure 1.3: Variation in the observed position of the source star due to the presence of a star ahead of it, which is in motion due the presence of an exoplanet. When the light is obstructed by the star-planet system lying in front of the source star, the star's light will bend over and creates an illusion that the star is at some other point. When the system lying ahead moves, its actual position can be see. A periodic position change of the star can thus be observed and whenever the obstruction happens, lensing phenomenon can be observed.*Source*: [40]
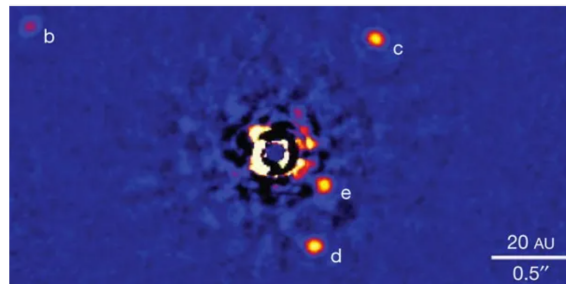
Figure 1.4: Direct imaging of HR8799. Source: [14]

can be removed optically by using instruments such as corona-graphs or while post-processing the image, where the starlight is detected and distinguished from that of the planet. Also, if the angular separation between the planet and the star is very small, then using this method becomes very difficult. This imaging technique is very preferable to planets with orbital radius greater than 5AU, planetary systems with fainter stars, young giant planets or if the stars are cooler ones. Even with the said limitations, this technique has been able to study some of the massive and self-luminous planets. By mid 2020, around 50 planets had been discovered by direct imaging. With more development in the technologies used, this technique has high prospects in the future [18] [50].

1.1.5 Astrometry

When we say that a planet is revolving around a star, what actually happens is that both the planet and the star are moving around the the center of mass and since the star has very high mass and size compared to that of the planet, the center of mass is more close to the star than to the planet. The motion of the star
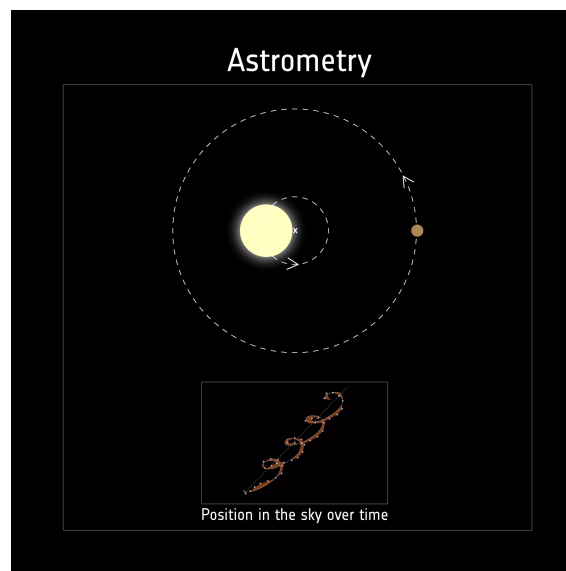
Figure 1.5: Illustration of how the position of the star revolving around its barycenter changes in the sky over time. Astrometry uses this variation in the star's position in the sky to detect the exoplanets. Source:

around the center of mass causes the star to have a wobble effect. In astrometry technique, the barycentric motion of the star is observed and its position relative to the background is measured. This technique mainly focuses on detecting planets that orbit their stars at a wider range. They are more sensitive to this section of planets as the displacement amplitude of the center of mass corresponding to these planets increase with the revolution period. This is why this technique detects planets of orbital period of several years.Astrometry is a very powerful method to detect exoplanets and complements many of the other methods but when compared to other techniques, this method has detected less number of planets. With the launch of Gaia satellite in 2013, the prospects of finding exoplanets using astrometry is promising [39] .

Each of these techniques have their own strengths and challenges and so it is not possible to select one from them and say that it is the best technique to detect an exoplanet. There are cases where it is possible to apply multiple detection techniques to detect the same planet. In such cases, using methods that complement each other will help in understanding different features of the planet.

1.2   Planet Types

As more number of exoplanets came to be known, their classification was necessary for the ease of study. The classification can be done on the basis of mass and size.Even though planets are classified on the basis of mass or size, different variety of planets can be found within each of these classifications. Planets can be broadly divided into four categories: terrestrial planets, Super-Earths, Neptune-like planets and Gas Giants.

1.2.1   Terrestrial Planets and Super-Earths

Exoplanets that have size between half the size of Earth and twice of Earth belongs to the group of terrestrials. Usually composed of rock, iron, silicate, water and sometimes even carbon, they are found to be dominated by the composition of rock or iron, leading to their name 'rocky planets' . They are found to have a solid or liquid surface.

These planets, if are present in the habitable zone, given the right atmospheric conditions, life could be found in them and can be classified into the group called 'potentially-habitable'. The first earth-size exoplanet in the habitable zone, Kepler 186f, was discovered in 2014 by NASA's Kepler and though many other planets similar to the size of Earth had been discovered, they were either too close to their stars or too far away from them, which cancels out the possibility of life present in them [41].

Terrestrial planets that are twice as the size of Earth or greater are called as Super-Earths.They could be up-to 10 times the size of Earth and yet, they tend to be lighter than planets such as Neptune.

1.2.2   Neptune-like planets

Neptune-like planets have size near to that of Neptune or Uranus. With Helium and Hydrogen being the major components in their atmosphere, these planets have a heavy, metal core [46]. These planets usually tend to have thick clouds, which prevents researchers from studying the signature elements of its atmosphere but in case the clouds doesnt block the view, like the HAT-P-11b, elements in the atmosphere can be studied[7] . Gliese 436 b, the first hot Neptune that has been detected and confirmed was detected using transit photometry. Planets with size falling between that of Super-Earths and Neptune are called as mini-Neptunes.

1.2.3   Gas Giants

These category of planets also have Hydrogen and/or Helium as the major components in their atmosphere.But unlike the terrestrial planets, these planets do not have a rocky core, instead has a solid core surrounded by gases. These giants are similar to the size of Jupiter and Saturn and they tend to produce a significant amount of wobbling to the stars around which they revolve. 51 Pegasi b, a hot-Jupiter and the first confirmed exoplanet, was detected by radial velocity technique in 1995.

By 2021, more than 5000 exoplanets have been detected using various of the previously mentioned techniques among which 35% are Neptune-like planets, 30% being Gas Giants, 31% are the 'mysterious' Super-Earths and 4% being Terrestrial planets[11]. Among them, transit method alone has contributed towards the detection of more than 3900 confirmed planets and radial velocity method to more than 900 [17] and both these methods have played a major role in detecting exoplanets and created major milestones along the way.

Radial velocity technique, being one of the first methods that played a milestone role in the detection of exoplanets, studying this method, trying to study star-planet systems with this technique and trying to reproduce their radial velocity curve is what is being tried to achieve through this project.

Chapter 2

Radial Velocity Curve

The radial velocity technique or the RV method is an indirect technique that is widely used to detect planets around other stars. With the detection of 51 Pegasi b, in 1995 by Mayor and Queloz [35], this method has helped in the detection of many exoplanets, this method has played a huge role in the field of exoplanets and still continues to provide huge contributions.

This method studies the Doppler shift observed in the spectrum of stars in order to detect whether there is a planet around them or not. In accord to this reason, this method is also sometimes known as the Doppler method.

2.1   Doppler Shift

Consider an observer A in a frame of reference A', at rest. There is another observer B in another frame B', with a torch of light. A and B are separated by a distance d. The light travels at the speed of $3 \times 10^8 m/s$ (denoted by c) and A observes the frequency of the light from B to be ν_0. Now suppose that B is moving at a velocity v away from A. After some time, if A tries to measure the frequency of light, since the light is approaching him from a much more longer distance than before, the frequency measured will be lower than ν_0, given by:

$$\nu_r = \nu_0 \times \sqrt{(1 - v/c)/(1 + v/c)} \tag{2.1}$$

Now, suppose that B is moving closer to A instead of receding with a velocity v. Then, A upon measuring the frequency of light will observe that the frequency has increased from ν_0, since in this case the light is travelling a much more shorter distance than before. Then, the new measured frequency can be given as :

$$\nu_a = \nu_0 \times \sqrt{(1 + v/c)/(1 - v/c)} \tag{2.2}$$

Similarly, when a star moves away from the point of observation, its frequency decreases and if it is moving closer, its frequency increases. In the electromagnetic spectrum, it can be seen that the colour red lies on the lower frequency side and the blue colour lies toward the higher frequency side. So, the receding frequency lies toward the red end of the spectrum and the approaching frequency lies toward the blue end of the spectrum. Due to this reason, while observing a star, if it is moving away, its spectrum is said to be "red-shifted" and if it is moving closer, the spectrum is said to be "blue-shifted"[6].
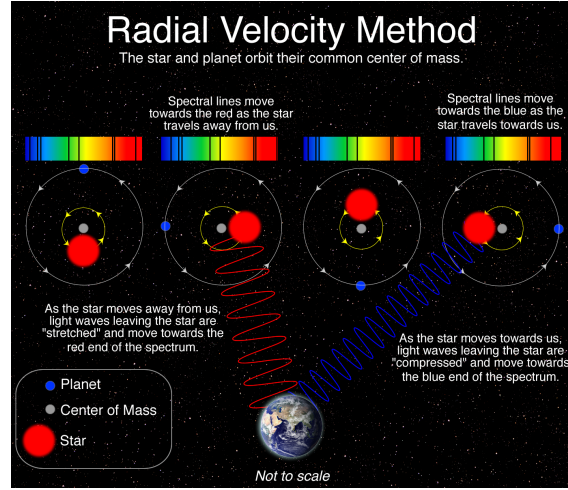
Figure 2.1: The red shift and the blue shift observed as the star travels away and towards the observer respectively. Source: [51]

Equations 2.1 and 2.2 can be written together as a single equation with respect to the corresponding wavelength λ of the light and the wave vector $\vec{k}$ as :

$$\lambda = \frac{\lambda_0 \times (1 + (\vec{k} \cdot \vec{v})/c))}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{2.3}$$

It can be seen from this equation that the Doppler effect depends on the projection of the velocity vector $\vec{v}$ along the line of sight, which is basically the radial velocity of the source. It can also be seen that there is a square of magnitude of the velocity vector v present in the denominator, but this term can be neglected considering that the velocity is very small in comparison to the speed of light [34]. Equation 2.3 provides a relation between the wavelength of the light and the velocity of the source and so it could lead to considering the radial velocity to be a function of time. By studying this radial velocity and how it varies, by studying the pattern, it will be possible to detect the presence of planets around the star and also other characteristics of that planet.

For radial velocities with low- amplitude variations, in consideration with the barycentric correction λ_B, the radial velocity can be written as :

$$\vec{k} \cdot \vec{v_s} = c \times \frac{\lambda_B - \lambda_0}{\lambda_0} \tag{2.4}$$

This radial velocity is not the absolute value of the radial velocity as determining absolute radial velocities by the Doppler effect is affected by many factors such as gravitational red-shift, line asymmetries, convective blue-shifts that are found in stellar spectral lines, etc. These effects contributes to an offset of about 1km/s in the radial velocity readings in the case of Sun-type stars and also for high resolution spectrograph, but they are almost constant with time [16][32].

2.2 Star-Planet System

In a star-planet system, both the star and the planet are moving around the barycenter. Upon visualising, it can be seen that the star is basically revolving around the

center of mass in a small circular orbit and the planet is revolving around the barycenter in a much bigger circular orbit. The motion will always be in such a manner that both the star and the planet stays at opposite ends of a straight imaginary line that connects both star and planet with the barycenter.When an observer studies this system, at the points where the planet is at its farthest or nearest (i.e., the star is nearest or farthest in this context) to the observer, the radial velocity he/she observes will be zero as at these points only tangential component of the velocity can be seen. If radial velocities are noted for this motion, from one of these said points to another, the radial velocity increases steadily, reaches a maximum and starts to decrease and reaches zero upon reaching the second point. Plotting these values with respect to time will give the radial velocity curve. For systems with eccentricity of the orbit equal to zero, this curve will be sinusoidal in nature. In cases where this eccentricity is not equal to zero, deviations from this sinusoidal shape are observed.

2.3   Orbital Parameters of a Star-Planet system

When an observer views the night sky from Earth, the plane that he/she sees the night sky to be in, is called as the "plane of the sky", a.k.a the "reference plane". Now, if there is a star in the sky that this observer is looking into, then the line that joins that observer and that object which he studies is called as the "line of sight". Now suppose that this particular star that the observer studies has a planet around it. Then, the plane at which this Star-Planet system exists is called as the "orbital plane". This "orbital plane" and the "reference plane" need not necessarily be lying parallel, they are inclined at some angle for many systems. This angle between these two planes is called as the "angle of inclination" (denoted by i). The points where these two planes meet are named as the ascending and the descending nodes, former named for being the one after which the planet move closer to the observer and for the latter, the planet moves away from the observer after passing the descending node. These two nodes are joined together by the "line of nodes". The angle between the reference direction and the direction to where the ascending node lies, is called as the "Longitude of ascending node", denoted by Ω.

In case the orbits are circular in nature, the planet will be at the same distance away from the star at all times. But in case of planets with elliptical orbits that have some eccentricity (usually denoted by the letter e), the distance between the star and the planet will vary. At times, the planet is closest to the star and at times it is at the farthest. In the former case, the planet is said to be at its "periapsis" and for the latter case, the planet is said to be at the "apoapsis". The average of the distance between these two points will give the "semi major axis", which is denoted by a. The orientation of this elliptical orbit in the said orbital plane is given by the "angle of periapsis" (ω). ω is always measured from the ascending node of the system to its periapsis point.

The position of the planet around the star is understood using an angle that the planet covers as it revolves. The value of this angle is zero degree when it is at the periapsis and it starts to increase and reaches 360 degrees as it completes one full revolution. This angle is called as the true anomaly, denoted by the letter θ. This angle says about the star's position as well as the planet's position around the barycenter at a given time. Figure 2.2 gives a visual idea about these parameters.
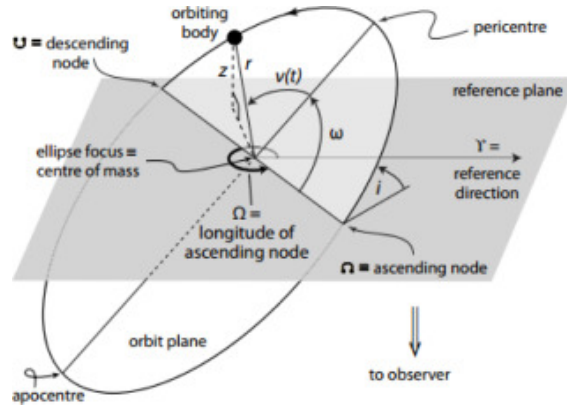
Figure 2.2: An illustration of the orbital elements of a star-planet system. Source : [25]

2.4   Deriving Radial Velocity

Suppose that the X-Y plane of the Cartesian coordinate system is considered as the reference plane, then the Z axis will be acting as the line of sight. Let the distance from the barycenter to the planet be denoted by $r(t)$ [36]. Then the distance of the star from the reference plane at the time t, denoted by $z(t)$, can be written in terms of $r(t)$, the true anomaly θ, inclination angle i and angle of periapsis ω as:

$$z(t) = r(t) \times sin(\theta + \omega) \times sin(i) \tag{2.5}$$

As the component of the velocity of the star that is parallel to the line of sight is the radial velocity, taking the time derivative of $z(t)$ will provide its value. Derivative of 2.5 with respect to time will give :

$$\frac{dz}{dt} = (\frac{dr}{dt} \times sin(\theta + \omega) \times sin(i)) + (r \times \frac{d\theta}{dt} \times cos(\theta + \omega) \times sin(i)) \tag{2.6}$$

Equation 2.6 gives the radial velocity, v_r. To express this equation without the derivative terms, consider that the orbit of the star whose semi major axis is denoted by a to have an eccentricity e, be expressed in the general form (in spherical polar coordinates) :

$$r = \frac{a \times (1 - e^2)}{1 + (e \times \cos \theta)} \tag{2.7}$$

Here, r will denote the points which will add up to an ellipse of eccentricity e when the θ changes. Taking the time derivative of equation 2.7 :

$$\frac{dr}{dt} = \frac{[(1 + (e \times \cos \theta) \times \frac{d}{dt}(a \times (1 - e^2)))] - [(a \times (1 - e^2)\frac{d}{dt}(1 + e \times \cos \theta)]}{[1 + e \times \cos \theta]^2} \tag{2.8}$$

This will give:

$$\frac{dr}{dt} = \frac{re(\frac{d\theta}{dt}) \times \sin \theta}{1 + (e \times \cos \theta)} \tag{2.9}$$

From Kepler's law of equal areas (ref.B),

$$\frac{dA}{dt} = \frac{1}{2} \times r^2 \times \frac{d\theta}{dt} \tag{2.10}$$

where A is the area of the ellipse. If P can be used to denote the period, then $\frac{dA}{dt}$ can be replaced by $\frac{A}{P}$. Then 2.10 can be written as

$$\frac{A}{P} = \frac{1}{2} \times r^2 \times \frac{d\theta}{dt} \tag{2.11}$$

Area of an ellipse is usually given by the equation $A = \pi \times (a \times b)$ where a denotes the semi-major axis and b denotes the semi-minor axis.

The eccentricity is generally given by the equation:

$$e = \sqrt{1 - \frac{b^2}{a^2}} \tag{2.12}$$

From this, semi-minor axis can be given as $b = \sqrt{a^2(1 - e^2)}$. Substituting these values to the equation 2.11 can be written as

$$\frac{\pi \times a^2\sqrt{1 - e^2}}{P} = \frac{1}{2} \times r^2 \times \frac{d\theta}{dt} \tag{2.13}$$

Upon rearranging,

$$r \times \frac{d\theta}{dt} = \frac{2\pi a^2 \times \sqrt{1 - e^2}}{r \times P} \tag{2.14}$$

From the general equation of the ellipse, substitution the value of r will give

$$r\frac{d\theta}{dt} = \frac{2\pi \times a \times (1 + e\cos\theta)}{P \times \sqrt{1 - e^2}} \tag{2.15}$$

Equation 2.15 in 2.9 will give

$$\frac{dr}{dt} = \frac{2\pi \times ae\sin\theta}{P \times \sqrt{1 - e^2}} \tag{2.16}$$

Substituting the values of $r\frac{d\theta}{dt}$ and $\frac{dr}{dt}$ in the equation of radial velocity (equation 2.6) will give

$$\frac{dz}{dt} = (\frac{2\pi \times ae\sin\theta}{P \times \sqrt{1 - e^2}} \times sin(\theta+\omega) \times sin(i)) + (\frac{2\pi \times a \times (1 + e\cos\theta)}{P \times \sqrt{1 - e^2}} \times cos(\theta+\omega) \times sin(i)) \tag{2.17}$$

Taking the common terms together outside and applying the trigonometric identities $\cos(\theta + \omega) = (\cos\theta \times \cos\omega) - (\sin\theta\sin\omega)$ and $\sin(\theta + \omega) = (\sin\theta \times \cos\omega) + (\sin\omega \times \cos\theta)$, the radial velocity v_r can be written as

$$v_r = \frac{2\pi \times a\sin i}{P \times \sqrt{1 - e^2}} \times [(\cos(\theta + \omega)) + (e \times \cos\omega)] \tag{2.18}$$

The term accompanying the triginometric terms is called as the semi-amplitude of the radial velocity or "radial velocity semi-amplitude". It is usually denoted by the letter k and when it is used to denote a star's semi-amplitude radial velocity, it is written as k_*. So,

$$k_* = \frac{2\pi \times a_* \sin i}{P \times \sqrt{1 - e^2}} \qquad (2.19)$$

where a_* denotes the distance of the star from the barycenter. Therefore,

$$v_r = k_* \times [\cos(\theta + \omega) + (e \times \cos\omega)] \qquad (2.20)$$

Equation 2.20 will give the radial velocity of a star for different values of θ. As θ values goes from 0°to 360°, one revolution will be complete. Radial velocity plots against the phase of the orbit will provide sinusoidal curves, even for orbits whose eccentricity is not equal to zero. Some of such radial velocity vs phase graphs which ranges from 0°to 720°for different periapsis angle values and eccentricity equal to 0 are shown in figures in appendix refappendix c. It can be seen that for the same value of eccentricity, as the value of ω changes, the maximum and minimum values of the radial velocity does not change but a shift in the radial velocity values can be observed as θ goes from 0°to 720°. As the eccentricity value increases, the radial velocity curve remains to retain a sinusoidal shape, the maximum and minimum values of the amplitude will increase but the curve will remain to be sinusoidal.

2.5 Radial Velocity Curve

A radial velocity curve is a plot of radial velocity of the star against the time at which these reading are taken. This curve will appear to be in a sinusoidal shape as long as the eccentricity is zero. But, in cases with non-zero eccentricity it can be seen that the shape of the curve is not exactly sinusoidal, instead the peaks takes place in an angle as shown in appendix D.

In case where the eccentricity is 0, the star as well as the planet will be moving at a constant angular velocity. The same amount of $d\theta$ will be covered in all dt times. But, when eccentricity is not zero, in accordance to the second law of Kepler's laws of planetary motion (Appendix B), the $d\theta$ values will vary for the same dt values in order to cover the same amount of area. The orbital velocity will not be a constant and so does the radial velocity. The velocity will decrease as the body goes farther from the barycenter and near that point the velocity will be higher. This can be seen in the radial velocity curves also. The region where shorter and narrow dips are seen denotes the velocities and time when the body is very much closer and the taller and wider peaks denote when it is farther away. The peaks get wider because in this region the body moves slowly and so it takes more time to cover. In regions where it travels fast, only little time is needed, which makes it narrow. This effect is more prominent when the value of eccentricity gets higher and higher.

Equation 2.20 provides the radial velocity in dependence to θ. So, while plotting the radial velocity, the time corresponding to the θ values need to be obtained and replaced in the x axis of the graph. If the mass of the star is M_* and mass of the planet is m_p and a_* and a_p denotes their separation from the barycenter respectively, then the orbital separation can be given as $a = a_p + a_*$. It is important to note that the total angular momentum will always be a conserved quantity. For an eccentricity e, this angular momentum can be given by the formula:

$$L_{total} = \sqrt{G \times M_* \times m_p^2 \times a(1 - e^2)} \qquad (2.21)$$

As from equation 2.21 it can be directly seen that since all the quantities are constants in the right hand side, the angular momentum is a conserved quantity as it'll be a constant.

For a stable system, both the planet and the star will be having the same angular velocity. Which means that both the star and the planet will be covering the same value of $d\theta$ for the same dt. So they will be in opposite sides of the center of mass at all times. The angular velocity and the total angular momentum are related by :

$$\omega_v = \frac{L_{total}}{m_p \times r^2} \qquad (2.22)$$

Substituting the value of total angular momentum from equation 2.21 in equation 2.22 will give the value of angular velocity. Now, since the angular velocity is given by:

$$\omega_v = \frac{d\theta}{dt} \qquad (2.23)$$

The value of dt corresponding to the $d\theta$ can be obtained by rearranging the equation 2.23 and substituting the value of angular velocity in it. In this way, the time corresponding to the θ values can be calculated and the radial velocity can be plotted using this time values in place of the phases and the radial velocity curve which depends on time can be obtained.

2.6 Analysing the Radial Velocity Curve

While studying the star-planet system, initially a high resolution echelle spectrograph is used to study the light coming from the star by splitting it up[5]. From its spectrum, spectral class can be identified, which will lead to the understanding of its luminosity. From that, mass of the star can be derived. By studying the shifts observed in the frequencies detected, the star's radial velocities are obtained.

By plotting the radial velocities against time, radial velocity curve is obtained, from which the length of one entire cycle will give the information regarding the period of the star (or the period of the planet). The amplitude of the curve can give the maximum radial velocity of the star. Orbital radius of the planet can then be obtained from the Kepler's third law, or the law of periods. In this way, analysis of radial velocity curve alone can give information about some of the important parameters related to the exoplanet.

2.7 Obtaining the Mass

One of the major advantages of the radial velocity method is that, this method can help in finding the minimum mass of the planet orbiting the star. The reason to why the calculation of the minimum mass is done and not the actual mass of the planet

will be discussed soon. From the Kepler's third law (ref B, the relation between the period of a planet and its orbit's semi-major axis can be given as :

$$P^2 = (a_p)^3 \times \frac{4\pi^2}{G \times (m_p + m_*}$$

(2.24)

where P is the period of the planet, a_p is the distance of the planet from the barycenter, G is the universal gravitational constant (G(magnitude) = 6.67×10^{-11} and units: $m^3kg^{-1}s^{-2}$), m_p denotes mass of the planet and m_* denotes the mass of the star.

In a two-body problem which involves two masses m_p and m_* positioned at distances a_p and a_* respectively from the barycenter,

$$m_p \times a_p = m_* \times a_*$$

(2.25)

From this,

$$a_* = \frac{m_p \times a_p}{m_*}$$

(2.26)

From equation 2.24 substitude the value of a_p in equation 2.26 and then substitute this value of a_* in equation 2.19 :

$$k_* = \frac{2\pi \times \sin i}{P \times \sqrt{1-e^2}} \times \frac{m_p}{m_*} \times \sqrt[3]{\frac{G \times (m_p + m_*) \times P^2}{4\pi^2}}$$

(2.27)

Upon simplifying,

$$k_* = \sqrt[3]{\frac{2\pi \times G}{P}} \times \frac{m_p \sqrt[3]{m_* + m_p}}{m_*} \times \frac{\sin i}{\sqrt{1-e^2}}$$

(2.28)

Since mass of the star m_* is very much greater than that of the planet m_p, $m_* + m_p$ can be approximated to m_* itself. then, equation 2.28 can be written as :

$$k_* = \sqrt[3]{\frac{2\pi \times G}{P}} \times \frac{1}{m_*^{\frac{2}{3}}} \times \frac{m_p \times \sin i}{\sqrt{1-e^2}}$$

(2.29)

By rearranging this equation, the mass of the planet m_p can be obtained. But, the angle of inclination is not a known value while studying the radial velocity of the star. If the inclination is 0°, then the star will be seen to be moving in the reference plane itself, due to which the Doppler effect cannot be observed. In order to see the frequency change in the light coming from the star, the star is supposed to be moving in a manner similar to a to and fro motion when an observer views the star from a point. For this to happen, the angle of incidence is supposed to be more closer to 90°than 0°. The difference in velocities can be found to be most prominent when the angle is 90°. Upon putting i = 90°, the mass derived from the equation 2.29 will be the minimum value as the $\sin i$ will be equal to 1. If $i < 90°$, then $\sin i < 1$ and so, as the mass of the planet is inversely related to $\sin i$, the mass will increase accordingly. As i is not a known value for the star-planet system in this method, mass of the planet is written in terms of $\sin i$ as :

$$m_p \sin(i) = \sqrt[3]{\frac{P}{2\pi G}} \times k_* \times \sqrt{1 - e^2} \times m_*^{2/3} \qquad (2.30)$$

The mass found using this method will always be smaller than the actual mass of the planet. But it is fairly a good estimate to the original mass since orbits rarely have large i values.

Thus, by studying the spectral lines of the star, by observing the shifts, radial velocity can be measured. From this radial velocity value, orbital period can be measured by calculating the time taken for the planet to go from $\theta = 0°$ to $\theta = 360°$. Once the time period is known, using Kepler's third law (ref B, the distance of the planet to the barycenter can be calculated. The total orbital separation can then be found upon finding the a_* value and by adding the two, $a = a_* + a_p$. Eccentricity of the orbit can also be obtained by running a fit on the data. Finally, minimum mass of the planet can also be derived from this method. In this way, radial velocity method helps in understand different properties of the star-planet system instead of a mere detection of an exoplanet.

Chapter 3

Modelling Synthetic RV curve

From the measured values of the radial velocity of a star, several parameters such as the orbital period, eccentricity, orbital distance of the planet, mass of the planet, etc. can be studied. From these known parameters, a radial velocity curve can be plotted as a function of time.

3.1 Code Design

3.1.1 Parameters

The input parameters required for modelling are :

1. **Mass of the Star** : The mass of the star is denoted in terms of solar mass (1 solar mass is equal to 1.98×10^{30}). There are different ways through which mass of a star can be calculated. One way is to use the relation that connects the mass and luminosity of the star, according to which the ratio of the luminosity of the star and the Sun is equal to some power a of the ratio of the masses of the star and the Sun respectively. The value of a depends on which category the star belongs to in a H-R diagram. These values have been calculated and published which are used for this project.

2. **Eccentricity of the orbit** : The eccentricity of the orbit describes its flatness. For eccentricity equal to zero, the orbit is circular in nature and the values between 0 and 1 corresponds to elliptical orbits. Eccentricity equal to unity corresponds to a parabola and greater than that to hyperbola. Planets that are orbiting the stars can have eccentricities less than 1 only. While calculating the eccentricity value, it is the eccentricity of the sun's orbit that is being calculated, and this value is the same for the corresponding planet as well.

3. **Semi major axis of the orbit the star takes around the barycenter**: This value is given in terms of the astronomical units. The sum of this value and the semi-major axis of the planet's orbit, which can be calculated using Kepler's third law, will give the orbital separation.

4. **Time period**: The time taken for the star to complete one complete revolution around the barycenter is the time period, which could be days or years. This value, if not in years, is converted to years before carrying out the calculations. This, though is a calculated value from observing the star, this value is same for the corresponding planet too.

5. **Angle of inclination**: The angle made by the orbital plane with the reference plane is the inclination angle but, this value is not a precisely known value when it comes to the radial velocity studies. So, if it is not known, a reasonable value can be given while carrying out the code.

6. Angle between the ascending node and the periapsis, a.k.a the angle of periapsis

The true anomaly as mentioned before is another parameter but it is set from 0°to 720°.

3.1.2   Creation and Execution of the model

The above mentioned parameters are derived from studying the data obtained from a star-planet system. The mass of the star is taken directly from the published values in order to execute the program.

The libraries used for this modelling are NumPy [21] for basic mathematical calculations, Astropy [4] for the usage of constants and units as well their conversions and Matplotlib [26] for plotting the radial velocity curve. Provision for the user to enter the parameters as given along with the units in which they are to be entered and these values are converted to SI units. Radial velocity equation given in equation 2.20, orbital separation which is obtained from the kepler's third law B, equation of the ellipse in the polar form 2.7, angular velocity 2.22 and dt values from 2.23 are defined. The phase is defined from 0°to 720°. $d\theta$ is given as 1, meaning the increment of angle increases by the value one. For zero eccentricity orbits, dt will be same for all 720 $d\theta$s taken for the star to complete two complete revolution. But if the eccentricity is not equal to zero, then the dt values will vary. So, using the function defined, all the dt values that will take place for two complete revolutions are calculated and stored in an array.The radial velocity is also calculated for the given parameters and stored in an array. The length of the array of both dt and radial velocity values stored should be the same in order to carry out the plotting. Then, from dt values, using a for loop time is obtained, which is another array of equal length but with dt values appended in a way to convert the difference in time into intial time to final time. Then, the radial velocity curve is plotted with time on the x-axis and radial velocity on the y axis. The graphs plotted for different eccentricity values and ω values are given in Appendix D. It can now be seen how different the graphs would be from that plotted with phase(degrees) in the x-axis as that shown in Appendix C.

After studying a RV data of a star-planet system, which is discussed in the following sections, the parameters can be obtained and used in this model to produce radial velocity curve.

3.2   Analysis of Radial Velocity Curve

Carrying out the analysis of the radial velocity curve as mentioned in section 2.6 is not as simple as that put in theory. There are multiple reasons to why direct plotting and analysing the raw data is difficult. The readings of the radial velocity is not always done at equal intervals of time and at times this gap between two adjacent readings can be very long that it gets difficult to observe a structure to the data plotted or visualize/have a curve pass through the said data points. The

number of stars that can be viewed at a time is very limited and a lot of stars need to be studied. This is one of the multiple reasons to why the radial velocity data is unevenly sampled. Another issue is that the signals received can contain noise and so signals need to be processed before they are analysed. Initially when the data is taken, the data is reduced, meaning calibrations are done to make the spectra to make it ready for the analysis. Usually bias correction is done to these images which is called as "debiasing" and by "flat-fielding", correction in pixel-to-pixel variations are done. All other cosmic rays can be removed from the spectra and other noises too, with the room for other corrections including high precise wavelength corrections[5][27]. Once these corrections are done, the spectrum of the star will be ready to be analyzed.

3.2.1 Periodogram Analysis

The obtained signal is then processed in order to detect the presence of an exoplanet. There are different signal processing methods available to study these signals such as Lomb-Scargle Periodogram, Pre-Whitening method, Keplerian Periodogram, Maximum-Likelihood periodogram, Bayesian analysis, etc. These techniques take in the radial velocity data as $x(t)$ where t is the timestamp of observations.

Periodogram analyses can be conducted in a given data to identify the dominant frequencies of the time-series. The frequency corresponding to the highest power relates to the time period of the star. When the data is unevenly sampled, Lomb-Scargle Periodogram is a very good technique to analyze the data. It has been used widely to study signals, especially the radial velocity data [44]. This method basically works like a Fourier transform. The given data is treated with a number of sines and cosine values across the frequencies which are taken from the given time values and their amplitude is reduced to fit the given data and the power is calculated as a result. The plot will be having frequencies on the x-axis and power on the y-axis. The maximum power is obtained at that frequency where a good match is obtained for the function with the entered radial velocity data. One of the biggest drawback put forward by this LS Periodogram is that, when the orbits have a good amount of eccentricity, this method fails to detect the signals properly. Also, this is designed mainly for systems which contains a single planet. In cases with many planets around the star, the data will naturally contain multiple signals, which this method miserably fails to detect. In such cases, methods like pre-whitening method can be used, which is similar to Lomb-Scargle method in terms of using sines and cosines and also in the Fourier domain, but is capable of detecting multiple signals in a given data. Another issue is that this method only considers the white noise whereas the light from stars often contains correlated noise.

Once the frequency corresponding to the maximum power is obtained by the periodogram analysis, taking inverse of the frequency will lead to getting the time period, which is probably given in days and needs to be converted into years in order to input in the model. Thus the second parameter for modelling can be obtained, first being the mass of star.

3.2.2 Folding Data

Once the period of the star P is known, the time-series data is folded to determine the next set of parameters. What is means by this is that the data that is lying

from, say 0 to t_n time, will be folded into a 0 to 1 and the plotting of the radial velocity will be done in 0 to 1 range over and over until all the points are plotted. The value of time in the range 0 to 1, called as the phase, is determined by the equation:

$$phase = \frac{time - T_0}{P - \frac{time-T_0}{P}} \tag{3.1}$$

This phase is not an angle representing 0 and 2π, instead it's just a number. If the plot is done with phase$\times$P on x-axis and folded radial velocity on y-axis, it will provide an idea on how the radial velocity of the star varies in one revolution.

3.2.3   Converting the rv-time graph into rv-phase graph

As mentioned in the previous sections, the radial velocity v/s time curve is not a proper sinusoidal curve when the eccentricity of the orbit is non-zero. Due to this reason, there is a need to convert the graph into one in which radial velocity v/s phase is given so that a sinusoidal shaped scattered data points can be obtained. This data can then be fitted to get the rest of the parameters. In order to do this conversion, Kepler's equation is taken into account[42][47].

For a planet that is moving in an orbit of eccentricity e with the star at one of the focii, if the period is denoted by P, semi-major axis by a and T is the time at which the planet reaches its point of perihelion, then the position of the planet at a later time t can be found using Kepler's equation. Things would have been very simple if e was always zero because in such cases, if the difference between t and T is equal to two-third of the time period, then it can be always said that the planet has covered two-third of the orbit starting from the perihelion passage. But since the orbits are mostly ones with some amount of eccentricity, this method of directly predicting the position of planet is not always possible. The true anomaly can help in understanding the position of the planet. In eccentric orbits also, this true anomaly will provide the position of the planet in that elliptic orbit. Now, if a circular orbit is visualized in the same plane with the semi-major axis of the elliptical orbit being its radius, and if a line is extended outward from where the planet is, then this line will intersect at some point of the circular orbit. It is to be noted that the circle passes through the perihelion and the aphelion of the ellipse. The angle between the perihelion and the point of intersection is called as the Eccentric anomaly, E. If E can be calculated, then variables like θ (true anomaly) can be obtained. Consider the equations:

$$r = a \times (1 - e\cos E) \tag{3.2}$$

and

$$tan\frac{\theta}{2} = \sqrt{\frac{1+e}{1-e}} \times tan\frac{E}{2} \tag{3.3}$$

The true anomaly M is given by the equation:

$$M = 2\pi \times \frac{t - T}{P} \tag{3.4}$$

The Kepler's equation:

$$M = E - (e \times sin(E)) \tag{3.5}$$

can be used to find the eccentric anomaly after calculating the mean anomaly by knowing T value. Once the eccentric anomaly is known, true anomaly values can be calculated.

The difficulty lies in the solving of Kepler's equation. Iteration methods can be used to solve the method or the Bessel's function can be made use of to find the solution. They will provide an approximate solution to Kepler's equation. Libraries that can solve the equation can also be used while coding to ease the work.

3.2.4   Fitting the Data to obtain other Parameters

Once the folding and conversion from time to angles are done, the next step is to fit a curve through these data points. One way of doing this is to apply least square fit to the set of data with the radial velocity equation derived before as the function to which fitting is to be done. Then, by keeping all the parameters locked in, slowly vary one parameter each to get the reduced χ-square value minimum. Reduced χ-square test is used to find the best fit to the given data. The ideal fit can be obtained when the value of reduced χ-square becomes unity. But, in cases where there are other unknowns, this value may not reduce to one itself. In such cases, the variation of parameter values need to be done until this value becomes a minimum. Here, since angle of incidence is not really a known quantity, semi-amplitude radial velocity, $k*$, itself is taken as one of the parameters for the fitting. The next parameter taken into consideration is the eccentricity of the orbit and the third parameter is the ω. The periapsis angle can vary depending on from where the reading are taken. But the semi-amplitude radial velocity and the eccentricity values will not change for the given star-planet system[15].

3.2.5   Finding Minimum mass and the orbital separation

Once the semi-amplitude raidal velocity, eccentricity and the ω values are obtained, the final step is to find the orbital separation and the minimum mass. For this, from the Kepler's laws in Appendix B, third law can be used. Then to find the minimum mass, from Section 2.7, equation 2.30 can be used.

3.2.6   Designing the code for the analysis

Lomb-Scargle periodogram from the time-series section of Astropy library is used to conduct the periodogram analysis on the given data. PyAstronomy library [12] is used to fold the data. Kepler[29], a python packaging is used to find solution to the Kepler's equation. lmfit [37] is the library used for modelling and finding the fit parameters. Then the mass of the planet, orbital separation and the distance of the planet from the center of mass is also calculated. The respective codes are given in Appendix A.
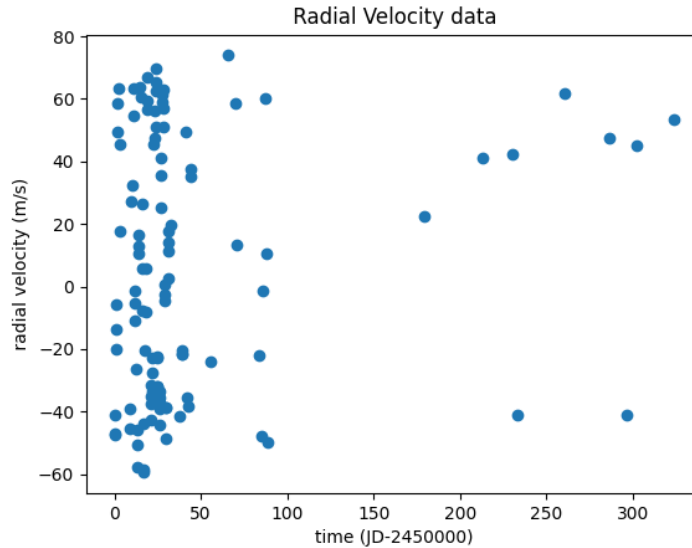
Figure 3.1: Plotting the radial velocity data with time on x-axis and radial velocity on y-axis

3.3   Studying 51 Pegasi

The first exoplanet detection and confirmation occurred in 1995 was that of the planet accompanying 51 Pegasi. Although a discovery of a companion to HD 114762 was made even before, it was then considered to be a brown-dwarf. The confirmation of an exoplanet was with the discovery of 51 Pegasi b, a Jupiter sized planet with a minimum mass of 0.46 $M_{Jupiter}$ which had a period of 4.2 days. The detection was carried out by the spectrograph called ELODIE, in France. There was a demonstration to show that the signal showed dynamical properties rather than a stellar phenomena. Following year, two more giant planets were announced and three more that are similar to 51 Peg planet in the next year. Since then, the detection has been ever so increasing.

The data for studying the radial velocity is sourced from NASA exoplanet archive [33] [10]. The data contains time as the first column, which is given in Julian Date. Then, the second column contains the radial velocity values in m/s units. The third column contains the error values in m/s. These columns are entered into arrays to make the calculation easy. The initial time is set to zero, which leads to the subtraction of that time from the following data points. Then, a radial velocity curve is plotted as shown in the figure 3.1.

From this it can be seen that the data points are distributed non-evenly over time and there are gaps between the observations. If a sine curve is tried to fit directly into this plot, one can get the image as shown in figure 3.2.

This image obtained is not easy to study and so there is a requirement for folding the data. From this plot, it is not possible to directly understand the period of the star (or the period of the planet) and so, periodogram analysis is applied and a graph as shown in the figure 3.3 is obtained.

The frequency corresponding to the maximum power is taken and its inverse gives the time period of the star. It is obtained to be 4.238 days, which upon converting to years will be approximately around 0.0116 years. Using this time period (in days),
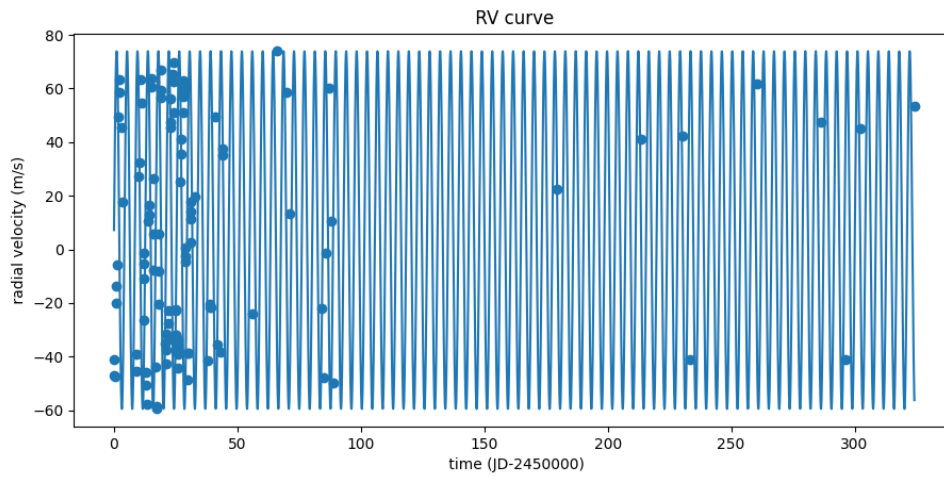
Figure 3.2: Trying to fit the radial velocity curve directly onto the data plotted.
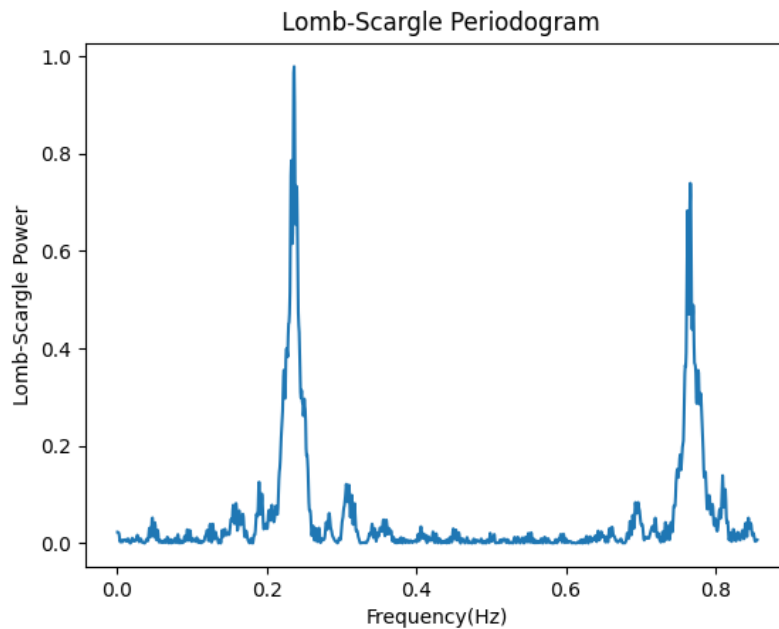

Figure 3.3: Applying Lomb-Scargle Periodogram to find the regions where signals have maximum strength to get the orbital period
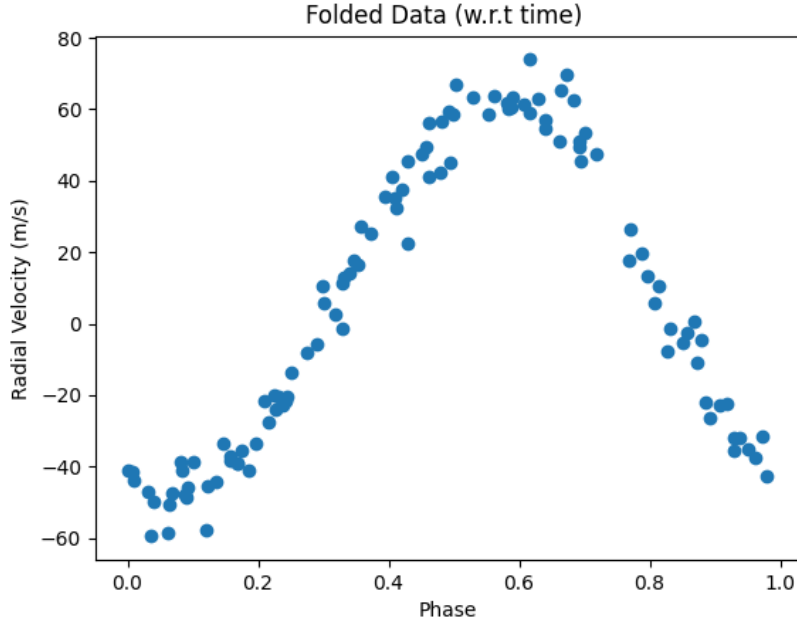
Figure 3.4: Folding the given time-series data with phase ranging from 0 to 1

the data is folded as shown in the figure 3.4.

It can be seen that the x-axis has points in the range of 0 to 1. This is called as phase but it does not represent the angles. Instead it's just a dimensionless number that represents the folded time. Upon multiplying this value with the period, the fold in terms of period can be observed.

But, this figure is in terms of time in x-axis and radial velocity in y-axis. In order to carry out further study, time needs to be converted into its corresponding angles. This is done by solving the Kepler's equation and then this angle values are used to plot the curve as shown in the figure 3.5.

Though these two figures 3.5 and 3.4 look similar in terms of sinusoidal shape, it is only because of the eccentricity being a small value. Using the lmfit least square fitting model which was defined using the radial velocity function, parameters such as k_*, e and ω are obtained with reduced chi-square value around 18. Though this is not an ideal value for the fit, this was as low as what the value went and so these parameters were taken.

The k_* was found to be 55.5m/s, e was found to be 0.024 and the ω value was found to be 17.1°. The actual values of these parameters are 55.9m/s, 0.012 and ω values can vary depending on from where the data is sourced. Since the eccentricity is having an error bound of 0.013, 0.024 lies within the desired limit. From this, minimum mass was calculated to 0.0471 $M_Jupiter$ and the orbital separation was found to be 0.0527AU, and these are in accordance to the published values. At the end, the radial velocity curve is plotted using the model as shown in the figure 3.6. In the similar manner, some more star-planet systems have been studied, and their parameters are also obtained which are given in the table3.1. The values taken along with their references are given in table 3.2.

From the parameters obtained in table 3.1, modelling of the radial velocity curves are done. HD 4308, a G-type main sequence star, has a low mass planet HD 4308
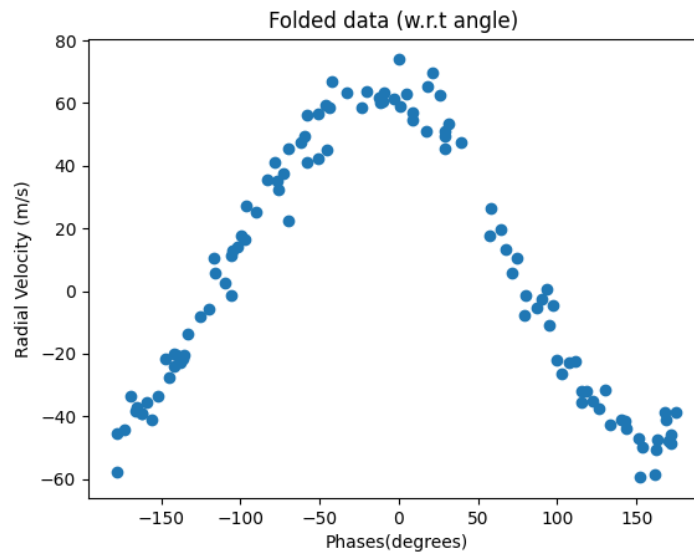
Figure 3.5: Radial velocity plotted against the phase (folded case). The perihelion is where the angle corresponds to zero, the point where the velocity will be maximum in nature.
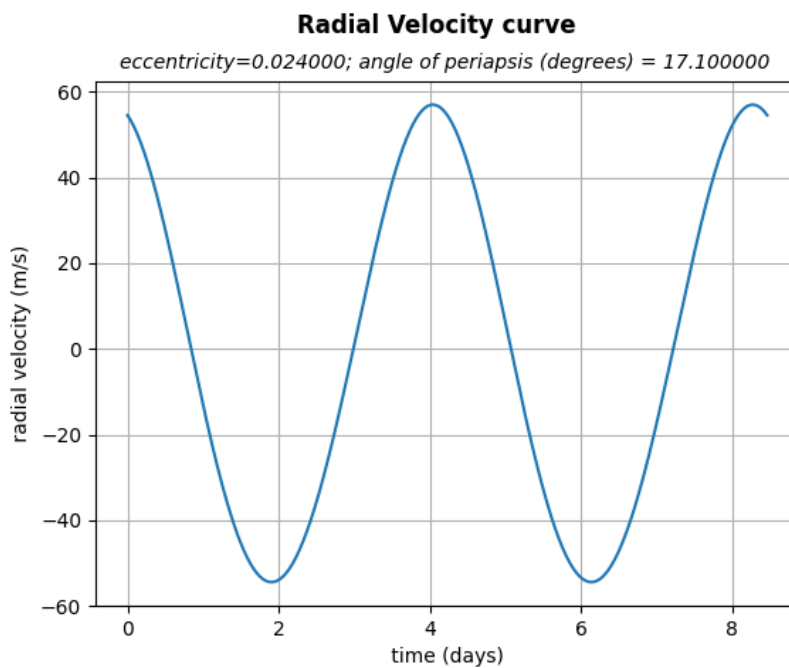

Figure 3.6: Radial velocity curve produced using the model based on the parameters studied for 51 Pegasi.

Parameters Obtained						
Planet	Orbital Separa- tion(AU)	Period (years)	Eccentricity	k_*(m/s)	minimum mass	ω —
51 Pegasi b	0.0527	0.0116	0.024	55.7	0.471	17.1
HD4308	0.12	0.041	0	4.1	0.047	89
HD4313	1.149	0.98	0.1	40.6	1.912	63

Table 3.1: Parameters obtained by studying the radial velocity data of the stars which have a planet around them. Using these parameters, radial velocity curve is produced with the help of the model produced.

Published Values						
Planet	Orbital Separa- tion(AU)	Period (years)	Eccentricity	k_*(m/s)	minimum mass	Source
51 Pegasi b	0.0527	0.0116	0.024	55.7	0.46	[1][2]
HD4308	0.12	0.041	0	4.07	0.044	[23]
HD4313	1.157	0.97619	0.147	40	1.912	[24]

Table 3.2: Parameters obtained by studying the radial velocity data of the stars which have a planet around them. Using these parameters, radial velocity curve is produced with the help of the model produced.

b whose orbit is having eccentricity essentially zero. HD 4313 is a K-type star, and the planet revolving around it is called as HD 4313 b, a giant whose mass is greater than that of Jupiter.
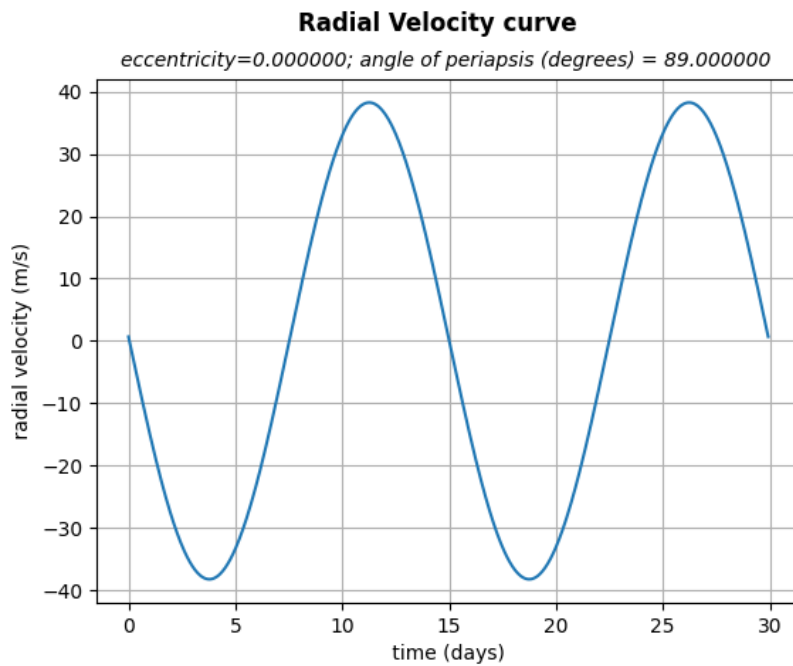
Figure 3.7: Radial velocity curve produced using the model based on the parameters studied for HD 4308.
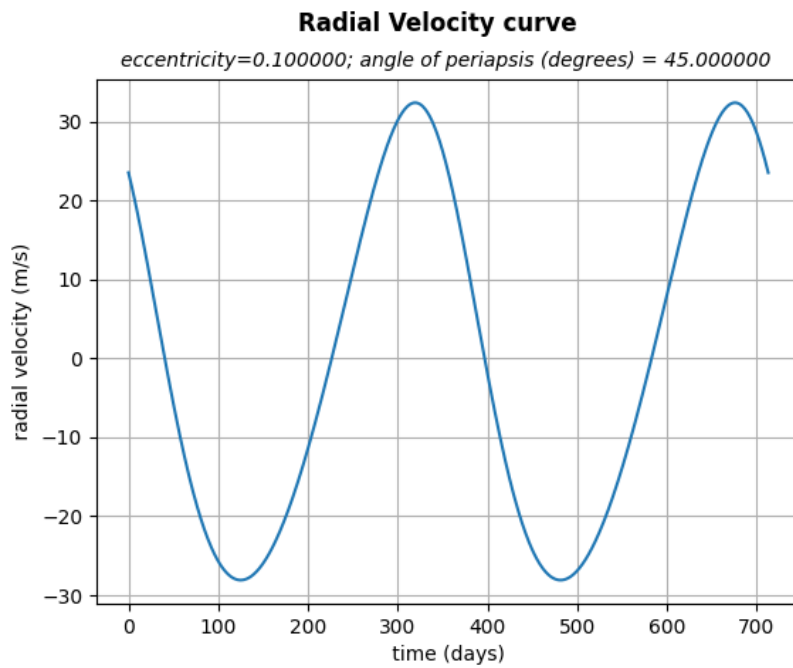

Figure 3.8: Radial velocity curve produced using the model based on the parameters studied FOR HD 4313.

Chapter 4

Conclusion

The radial velocity technique was the first successful method in detection of exoplanets and has proven to remain as very strong ever since. The success of this method is mainly contributed by the extremely sensitive spectrographs, which assists in the detection of even the slightest movement of the star by studying the variations in the stellar spectrum. These sensitive spectrographs allows the study to be carried out from the ground itself and since the stars need not be monitored continuously, this way of ground based detection is an advantage. This is not possible for transit photometry and this provides the former an edge over the latter technique.

4.1 Challenges Faced by Radial Velocity

As mentioned in the first chapter, every method has their own advantages as well as its own challenges. One of the major issues faced by the Doppler technique is that it cannot be used to find the actual mass of the planet. In most of the observed star-planet systems, the inclination angle is in such a way that the system is neither edge-on nor face-on. So, only that component of the velocity that is directed towards the observer is measured, leading to a minimum mass of the planet, which is leas that its true mass. At times, the mass of the planet could be much larger than the estimated value. Finding the minimum mass can provide an insight on how that planet is but the possibility that at least few of these could be low-mass stars is not a possibility that can be completely excluded.

Another challenge is something that has been discussed before. The case when angle of incidence is 0°. In other words, when the star-planet system is face-on, the star will not be moving away or toward the observer in a periodic manner, instead it will be seen to be moving in that plane itself, which will not lead to any changes in the spectrum observed.

Stellar activity also affects this method. Stellar activities in stars leads to bright and dim patches in them and as they rotate, the patches become visible to the observer and then disappears periodically. The bright patches lies toward the blue and the dim towards the red. This blue shift and red shift can give the false impression of the star moving closer and farther due to the presence of a planet. So, this can lead to false detection[9].

It is also to be taken into consideration that while transit method can be used to study many stars at a time, radial velocity study is limited to observing 30-40 stars

at a time. So the number of study at a moment is largely small in comparison to the transit method.

There are also other drawbacks such as the issues related with the p-mode oscillations, granulation and supergranulation , etc. Variation in the refractive index of the air can also affect the readings so the detectors need to be of high accuracy in order to sustain/overcome the the errors created by this variation. There are also other instrumental challenges such as the thermal and mechanical effects, slit illumination etc, caliberation of the wavelength and so on.

Though the above mentioned are few of the challenges faced by the Doppler technique, amid these issues, this method remains to be one with the second highest number of planet detections.

4.2   Curves produced by the RV Model

Modelling a radial velocity curve for a star-planet system can provide an insight on how the planet moves around the star. In this project,the radial velocity curve for stars with exoplanets are produced from the model created. The radial velocity curve obtained using this model are in favour of the radial velocity curve obtained from the data collected. For star-planet systems with significant eccentricities, the curve is widened at some regions and narrow at others, indicating the regions where the motion is slow and fast respectively. For zero eccentricity and for cases with extremely small eccentricities, the curve obtained appears to be proper sinusoidal in shape. Thus from viewing the graph produced, one can understand about the eccentricity based on the shape of that graph, the amplitude can give the maximum velocity, period can be understood from one cycle.

Appendix A

Codes

A.1   Radial Velocity v/s Phase

Code for producing the radial velocity v/s phase graphs shown in Appendix C is given below. For the particular set of graphs, the angle of incidence is given 90°, distance to the barycenter from the star is 0.0005AU, period is 5 years and mass of the star is given as 1 solar mass. These values are fixed and can be changed in the code as requirement arises. While running the code, user will be given a window to enter eccentricity as well as the angle of periapsis value. Thus, by using a fixed set of data, by varying eccentricity and angle of periapsis, the variation of radial velocity with respect to phase can be studied.

```python
# to produce radial velocity curves for the given data

# variables assigned :- eccentricity - e ; orbital period - p ;
    Semi major axis - a ; angle of inclination - i ; angle of
    periapsis - w ; true anomaly - theta ; Mass of star - m

import numpy as np
import matplotlib.pyplot as plt

# given data:
I = 90     # angle of inclination in degrees
A = 0.005  # semi major axis in AU
P = 5      # orbital period in years
M = 1      # mass of star in solar mass

# converting given data to standard values :
i = np.radians(I)   # converting angle of inclination from degrees
    to radians
a = A * 149597870.691   # converting semi major axis from AU to km
p = P* 365 * 24 * 3600  # converting orbital period from years to
    seconds

# inputting values for e and w :
E = input ( ' Eccentricity of the orbit : ' )
e = float ( E )
W = input ( ' Angle of periapsis : ' )
w = np.radians( float(W) )

#defining true anomaly:
theta = np.arange(0,721)
f = theta.astype(int)
```

35

```
28
29  # defining radial velocity:
30  def rv ( i, a, p, e, w, f):
31      nb = np.radians(f)
32      pa = 2 * np.pi * a * np.sin(i)
33      pb = p * np.sqrt(1- (e*e))
34      pc = np.cos(nb + w )
35      pd = e * np.cos(w)
36      rvsam = pa/pb
37      rv = rvsam * ( pc + pd) * 1000
38      return rv
39
40
41  # assigning function:
42  rad = rv ( i, a, p, e, w, f)
43
44
45  #plotting graph:
46  plt.plot(theta, rad)
47
48
49  #labelling graph:
50  plt.xlabel('phase ( degrees )')
51  plt.ylabel('radial velocity (m/s)')
52  plt.suptitle('Radial Velocity curve', fontweight = 'bold')
53  plt.title(' eccentricity = __; angle of periapsis = __ degrees ',
            style = 'italic',size = 10)
54  plt.show()
```

A.2   Plotting Synthetic Radial Velocity curve

The codes provides a synthetic radial velocity curve for a Star-Planet system. It is designed in a way that the user can input all the necessary parameters that are discussed in the third chapter. The phase dependence in the previous code is replaced by time in this code as mentioned. The rv curve is plotted against time as the result.

```
1   # to plot a radial velocity graph with time
2
3   import numpy as np
4   import astropy
5   from astropy import units as u
6   from astropy import constants as cons
7   import matplotlib.pyplot as plt
8
9   ms = float(input(' Mass of the star in solar mass : '))   # mass of
        the star
10  e = float(input(' Eccentricity of the orbit : '))      #eccentricity
        of the orbit
11  As = float(input(" Semi major axis of the star's orbit in AU: ")) #
        semi major axis of the star
12  P = float(input(' Orbital period of the star in years : '))   #
        orbital period
13  Id = float(input(' Angle of inclination in degrees : '))     #
        inclination angle
14  pe = float(input(' Angle of periapsis in degrees : '))     #angle of
        periapsis
15  ta = np.arange(0,721)              #true anomaly
```

```python
tr = ta.astype(int)

mstar = ((ms*u.M_sun).to(u.kg)).value          #mass of star in kg
astar = ((As*u.au).to(u.m)).value              #semi major axis of star'
    s orbit in m
p = P * 365 * 24 * 60 * 60              #orbital period in seconds
i = np.radians(Id)                     #inclination angle in radians
peri = np.radians(pe)                   #angle of periapsis in radians
f = np.radians(tr)                     #true anomaly in radians

def radvel(astar, i, p, e, peri, f):
  aa = 2 * np.pi * astar * np.sin(i)
  ab = e*np.cos(peri)
  ac = np.cos(peri+f)
  ad = p * np.sqrt(1 - e**2)
  ae = ab + ac
  rv = ( aa * ae )/ad
  return rv
vr = radvel(astar, i, p, e, peri, f)

def orbsep(p, mstar):
  G = (cons.G).value
  o = p/(24*60*60)
  oa = G * mstar * o * o
  ob = 4 * np.pi * np.pi
  oc = oa / ob
  od = np.cbrt( oc )
  return od
a = orbsep(p, mstar)

def ellipse( a, e, f ):
  ea = a * (1-(e*e))
  eb = 1 + (e * np.cos(f))
  ec = ea / eb
  return ec
r = ellipse( a, e, f)

def angularvel( r, a, e, mstar):
  G = (cons.G).value
  ia = G * mstar * a
  ib = 1 - e**2
  ic = ia * ib
  ie = r * r
  ig = np.sqrt(ic) / ie
  return ig
wv = angularvel( r, a, e, mstar)

def dtime(wv) :
  dth = 1
  dtheta = np.radians( dth )
  dt = dtheta / wv
  return dt
dt = dtime(wv)
time = [0]
for x in range(0,721):
  y =   time[x] + dt[x]
  time.append(y)
time.pop()

plt.plot(time, vr)
```

```
75 plt.xlabel(' time (days) ' )
76 plt.ylabel(' radial velocity (m/s) ')
77 plt.title(f"eccentricity=%f" % e +"; angle of periapsis (degrees) =
      %f "%pe, style = 'italic' , size = 10 )
78 plt.suptitle('Radial Velocity curve' , fontweight = 'bold')
79 plt.grid()
80 plt.show()
```

A.3   Studying Radial Velocity Data

The following two helps in conducting a periodogram analysis on the given radial velocity data, to fold the given time series data based on the time period obtained using the periodogram analysis, to plot radial velocity curve v/s phase(degrees) and to fit the curve to the data and obtain the parameters.

```
1 #to fit the data using periodogram and study the data
2
3 #importing libraries
4 import numpy as np
5 import matplotlib.pyplot as plt
6 from astropy import units as u
7 from astropy import constants as con
8 from astropy.timeseries import LombScargle as lst
9 from PyAstronomy.pyTiming import pyPeriod
10 import statistics as s
11 from PyAstronomy.pyasl import foldAt
12
13
14 #calling files and importing data
15 file_name = np.genfromtxt(fname='file_name.txt')
16 time = [0]
17 vel = [0]
18 err = [0]
19 for i in range(len(file_name)):
20   t1 = file_name[i][0]-file_name[0][0]
21   v1 = file_name[i][1]
22   e1 = file_name[i][2]
23   time.append(t1)
24   vel.append(v1)
25   err.append(e1)
26 time.pop(0)
27 vel.pop(0)
28 err.pop(0)
29
30 #applying Lombscargle periodogram
31 frequency, power = lst(time, vel).autopower()
32 ymax = max(power)
33 xpos = np.where(power==ymax)
34 xm = frequency[xpos[0]]
35 xmax = xm[0]
36 print('the frequency corresponding to the maximum power ',ymax,' is
      ',xmax,'Hz')
37 print('the time period is ',1/xmax,' days ')
38
39 #folding data and plotting it
40 phases = foldAt(np.array(time),1/xmax)
41 sorti = np.argsort(phases)
42 phases = phases[sorti]
43 velo = np.array(vel)
```

```python
44  velo = velo[sorti]
45  plt.figure('folded data')
46  plt.scatter(phases,velo)
47  plt.suptitle(' Radial velocity data')
48  plt.title('folded data ')
49  plt.xlabel('phase')
50  plt.ylabel('velocity(m/s)')
51
52
53  #plotting radial velocity data
54  #plt.figure('Radial velocity data')
55  plt.figure('rv data')
56  plt.scatter(time, vel)
57  plt.xlabel('time (JD-2450000)')
58  plt.ylabel(' radial velocity (m/s) ')
59  plt.title(' Radial Velocity data ')
60
61  #lombscargle graph plot
62  #plt.figure('Lomb-Scargle Periodogram ')
63  plt.figure('l-s Periodogram')
64  plt.plot(frequency, power)
65  plt.xlabel('Frequency(Hz)')
66  plt.ylabel('Lomb-Scargle Power')
67  plt.title(' Lomb-Scargle Periodogram')
68  plt.show()
```

```python
1   # To create a fit and obtain the best fit datas
2
3   #import libraries
4   import numpy as np
5   import matplotlib.pyplot as plt
6   from astropy import units as u
7   from astropy import constants as con
8   from astropy.timeseries import LombScargle as lst
9   from PyAstronomy.pyTiming import pyPeriod
10  from PyAstronomy.pyasl import foldAt
11  from scipy import optimize
12  from lmfit import Model
13  from scipy.special import jv
14  import kepler as kp
15
16
17  #calling files and importing data
18  file_name = np.genfromtxt(fname = 'file_name.txt')
19  time = [0]
20  vel = [0]
21  err = [0]
22
23  for i in range(len(file_name)):
24    t1 = file_name[i][0]-file_name[0][0]
25    v1 = file_name[i][1]
26    e1 = file_name[i][2]
27    time.append(t1)
28    vel.append(v1)
29    err.append(e1)
30
31  time.pop(0)
32  vel.pop(0)
33  err.pop(0)
34
35
```

```python
36 #applying Lombscargle Periodogram
37 frequency, power = lst(time, vel).autopower()
38 ymax = max(power)
39 xpos = np.where(power==ymax)
40 xm = frequency[xpos[0]]
41 xmax = xm[0]
42 tp = 1/xmax
43
44 print(' Frequency corresponding to the maximum power ', ymax,' is '
      , xmax,' Hz')
45 print('The time period is ', tp,' days')
46
47
48 #folding data
49 phases = foldAt(np.array(time),tp)
50 sorti = np.argsort(phases)
51 phases = phases[sorti]
52 velo = np.array(vel)
53 velo = velo[sorti]
54
55
56 #radial velocity curve - defining
57 def rv(x,e,kstar,w):
58   ab = np.cos(np.radians(x+w))
59   ac = e*np.cos(np.radians(w))
60   ad = kstar* (ab +ac )
61   return ad
62
63 #converting phases from 0 - 1 to 0 -time period
64 time = phases * tp
65
66 # finding the mean anomaly, M = (t-T)2pi/P
67 Vmax = max(velo)
68 x = np.where(velo == Vmax)
69 Tperi = time[x]
70 dt = time - Tperi
71 M = dt * 2* np.pi * xmax     #mean anomaly
72
73 plt.figure('Time -Folded Data')
74 plt.scatter(phases,velo)
75 plt.xlabel('Phase')
76 plt.ylabel('Radial Velocity (m/s)')
77 plt.title('Folded Data (w.r.t time) ')
78 # Finding eccentric anomaly by solving kepler's equation
79 e = .34
80 eccentricanomaly = kp.solve(M,e)
81
82 # finding true anomaly to the said time values
83 ta = np.tan(eccentricanomaly/2)
84 tb = np.sqrt((1+e)/(1-e))
85 theta = 2 * np.arctan(ta*tb)    # true anomaly in radians
86
87
88
89
90
91
92
93 #rearranging arrays to keep positive angle to one side
94 x = np.degrees(theta)
```

```
95  y = velo
96  va = np.array([np.where(x==0)])
97  va = va[0][0][0]
98  for i in range(va):
99    if x[i]>0:
100     newx = x[i]
101     newy = y[i]
102     x = np.append(x,newx)
103     y = np.append(y,newy)
104 for i in range(va):
105   if x[i]>0:
106     x = np.delete(x,0)
107     y = np.delete(y,0)
108
109 x=x
110 y=y
111
112 #applying least square
113 gmodel = Model(rv)
114
115 params = gmodel.make_params()
116 params.add(name = 'e',value = .34, min = 0, max = 360, vary = False
        )
117 params.add(name = 'kstar',value =55.7,min = 0, max = 70 ,vary =
        False)
118 params.add(name = 'w',value = 17.1, min = 0, max = 90, vary = False
        )
119
120
121
122 result = gmodel.fit(y,params,x=x)
123 print(result.fit_report())
124
125
126 #plotting graphs
127 plt.figure('Folded data')
128 plt.scatter(x,y)
129 plt.xlabel('Phases(degrees)')
130 plt.ylabel('Radial Velocity (m/s)')
131 plt.title('Folded data (w.r.t angle) ')
132
133 plt.figure('Fit data')
134 plt.scatter(x, y)
135 plt.plot(x,result.best_fit, '--',color = 'green', label = 'best fit
        ')
136 plt.xlabel('Phases(degrees)')
137 plt.ylabel('Radial Velocity (m/s)')
138 plt.title('Radial Velocity Curve ')
139
140 plt.legend()
141 plt.show()
```

A.4   Obtaining the Minimum Mass

The parameters obtained from the previous code is used to calculate the minimum mass of the planet. The parameters are entered to the code which upon running will directly give the orbital separation, semi-major axis of the planet's orbit in AU and the minimum mass in terms of Jupiter mass.

```python
# to find mass, k* and other parameters

#import libraries
import numpy as np
from astropy import units as u
from astropy import constants as c

# entering known values
p = 0.0116
p = p*365*24*3600

M = 1.11
m = (M*u.M_sun).to(u.kg)

# finding orbital separation a, a = a* + ap --- unit : AU
def orbsep(p, m):
  G = (c.G).value
  aa = p**2 * G * m.value
  ab = 4* np.pi**2
  ac = np.cbrt(aa/ab)
  od = (ac*u.m).to(u.au)
  return od
a = orbsep(p, m)
print('Orbital separation, a =',a)

ast = 0.000869
ap = a.value - ast
print('distance of the planet from the center of mass, ap =',ap*u.
    au)


e= 0.024
ks = 17.1

#mass --- unit : jupiter mass
def mass(ks, p,e,m):
  aa = (m.value)**(2/3)
  ab = ks * np.sqrt(1-e*e)
  ac = 2*np.pi*c.G.value
  ad = np.cbrt(p/ac)
  ae = ad*ab*aa
  return ae

mss = mass(ks,p,e,m)
mss = (mss*u.kg).to(u.M_jup)
print('m2sin(i) = ',mss)
```

Appendix B

Kepler's Laws of Planetary Motion

In the early 1600s, a German mathematician (and an astronomer) Johannes Kepler provided a set of three simple laws, which could explain the motion of planets. Although initially he was trying to provide a simpler description within the framework of Copernican model, towards the end he had to leave the idea of all orbits being circular in nature,which lead to greater simplicity in the explanation. He determined the shapes of orbits of different planets by triangulation of points that lie on the Earth's orbit. He conducted multiple observations at different times of the year. By observing the planets for many nights continuously, he was able to get an idea of the speeds with which they moved. These lead to summarizing the planetary motions into a set of three simple and neat laws, which are now known as the **Kepler's laws of planetary motion**[8][38][43].

Kepler's First law

This law discusses about the shape of the orbits in which the planets revolve around the star. Also known as the law of orbits, in the simplest terms this law can be explained as :

"Planets revolve around the sun in elliptical orbits with Sun at on of the focii"

Copernican model considered the orbits of the planets around the sun to be circular. Kepler said that these orbits are elliptical in nature. This means that the distance between the sun and the planet does not remain same instead they continue to vary over the period of time.
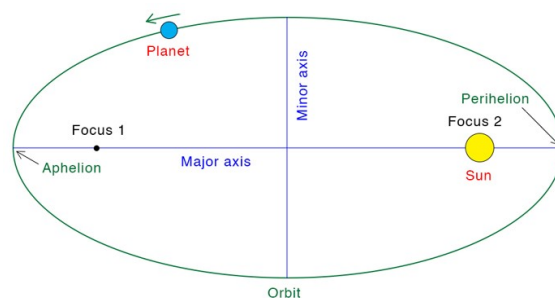

Figure B.1: An elliptical orbit taken by a planet around the sun. The sun is at one of its focii. Source: [31]

43

An ellipse has two points in it, in such a way that the sum of the distance to these points from any points that lie of the ellipse are same. these points are called the focii (singular: focus). The flatness of the ellipse is described by the parameter called eccentricity. The more the eccentricity is, the flatter will be the ellipse. Fig.B.1 shows how a planet revolves around with sun positioned at one of the focus points.

Kepler's Second Law

This law focuses on the speed of the planet at different points of the orbit. It is also known as the "Law of equal areas" In simplest terms, it can be said as:

"The velocity of the planet varies in such a way that a line joining the planet to the sun will sweep out equal areas in equal intervals of time."

Another phrasing is:

"The velocity of the planet will vary inversely proportional to its distance from the star"

There are different ways in which this law has been paraphrased from its original form published in "Astronomia Nova" in 1609 [3].

What this law essentially means is that, the area that is being swept out by an imaginary line that connects the sun and the planet is always same for equal intervals of time. Since the planet is not in a circular orbit ($e = 0$) instead elleptical($0 < e < 1$), and since the sun is at of the focii, the distance between the sun and the planet does not remain constant. At the point called perihelion (figB.1), the planet is closest to the sun and at the point called aphelion, the planet is at its farthest distance from the sun. Now suppose that the planet sweeps an area "A" in a given time interval "t".At some region that lies between the aphelion and the perihelion, as shown in the figB.2, the planet covers an area "A" in time "t" with a velocity v. In the region that covers aphelion, since the distance is longer, the in order to cover the same amount of area for a given period of time, the planet will have to travel slower. In the region which contains the perihelion, the distance to the sun is smaller and so in order to cover the same area in the given period of time, the planet will have to travel faster. So, the planet will travel at a higher speed when it's near the sun and at a lower speed when it is far away. For an area dA, if an angle θ is covered, dA can be given as :

$$dA = \frac{1}{2}\vec{r} \cdot \vec{r}d\theta \tag{B.1}$$

where $|\vec{r}|$ denotes the distance between the point when the planet is and the sun. So, the areal velocity will be,

$$\frac{dA}{dt} = \frac{1}{2}\vec{r} \cdot \vec{r}\frac{d\theta}{dt} \tag{B.2}$$

This can also be written as :
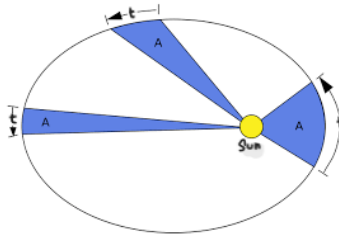
$$\frac{dA}{dt} = \frac{\pi ab}{T} \tag{B.3}$$

Figure B.2: A simple illustration of sweeping equal areas A in equal intervals of time t. Source:[30]

where, a and b denotes the semi-major and semi-minor axes of the ellipse and T denotes the total time period.

Kepler's Third Law

Known also as the "Law of periods", this law talks about how the semi-major axis of the planet's orbit can help in determining the orbital period of the planet.

"The square of the orbital period of a planet is directly proportional to the cube of the

For a planet for which the semi-major axis is a_p around Sun (can be generalized to any star) of mass M_*, the period P can be given as :

$$P^2 = \frac{4\pi^2}{G \times M_*} \times a_p^3 \tag{B.4}$$

By this law, the period of the planet increases rapidly as the distance between the planet and the sun increases. Using this, Kepler found that Mercury takes nearly 88 days, Venus 225 days, Earth 365 days and so on, up-to Saturn, which takes 10,759 days to revolve around the Sun.
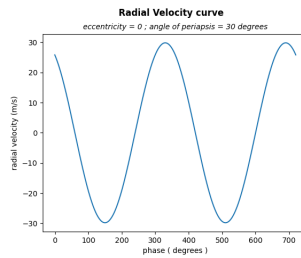
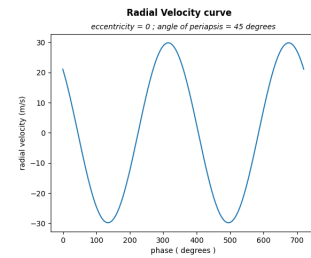# Appendix C

Radial Velocity v/s Pahse Graphs

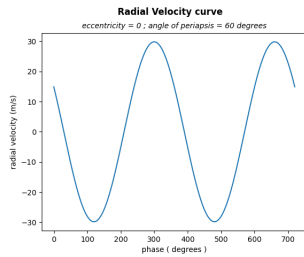The following graphs represent the radial velocity curve w.r.t phase ranging from 0°to 720°.

(a) $e = 0; w = 0°$
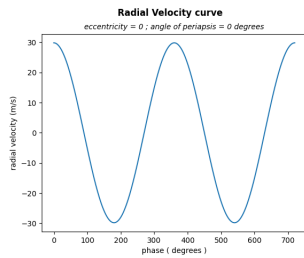
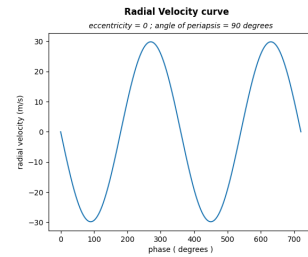(b) $e = 0; \omega = 30°$

(c) $e = 0; \omega = 45°$

(d) $e = 0; w = 60°$

(e) $e = 0; \omega = 90°$

Figure C.1: Radial velocity curves w.r.t phase for different values of ω for eccentricity = 0

For the same value of eccentricity, with difference in the value of periapsis angle, a shift can be observed. For the same periapsis angle, for different eccentricities, amplitude values gets more positive but the sinusoidal shape remains unchanged. But, radial velocity v/s time plots does not produce similar shapes for eccentric orbits. This can be seen in the following section.
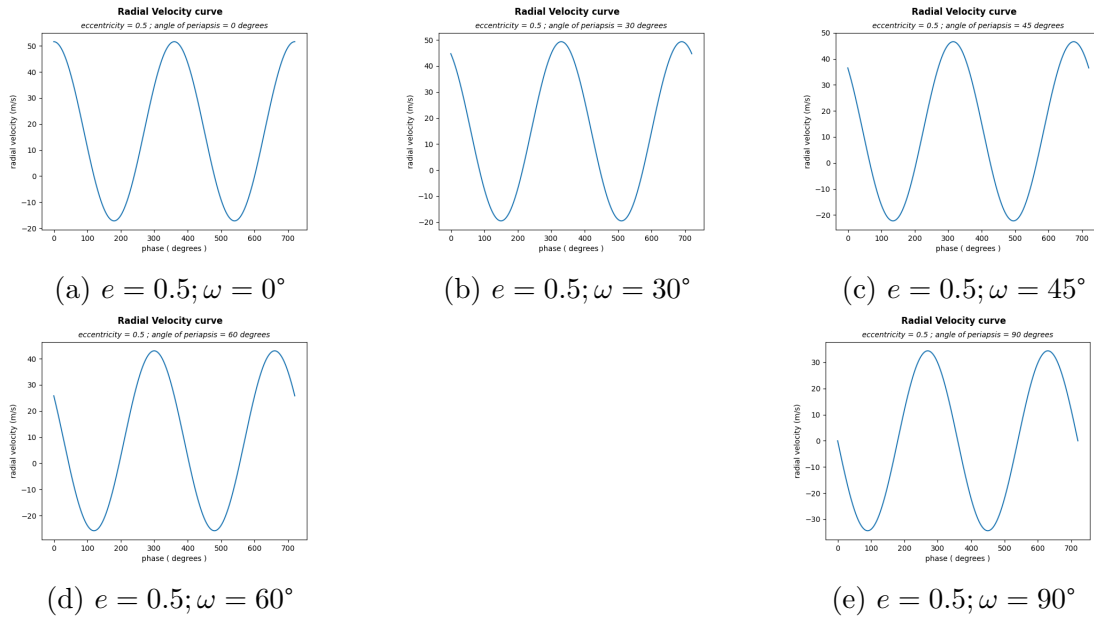
(a) $e = 0.5; \omega = 0°$ (b) $e = 0.5; \omega = 30°$ (c) $e = 0.5; \omega = 45°$

(d) $e = 0.5; \omega = 60°$ (e) $e = 0.5; \omega = 90°$

Figure C.2: Radial velocity curves w.r.t phase for different values of ω for eccentricity = 0.5

(a) $e = 0.9; \omega = 0°$ (b) $e = 0.9; \omega = 30°$ (c) $e = 0.9; \omega = 45°$

(d) $e = 0.9; \omega = 60°$ (e) $e = 0.9; \omega = 90°$
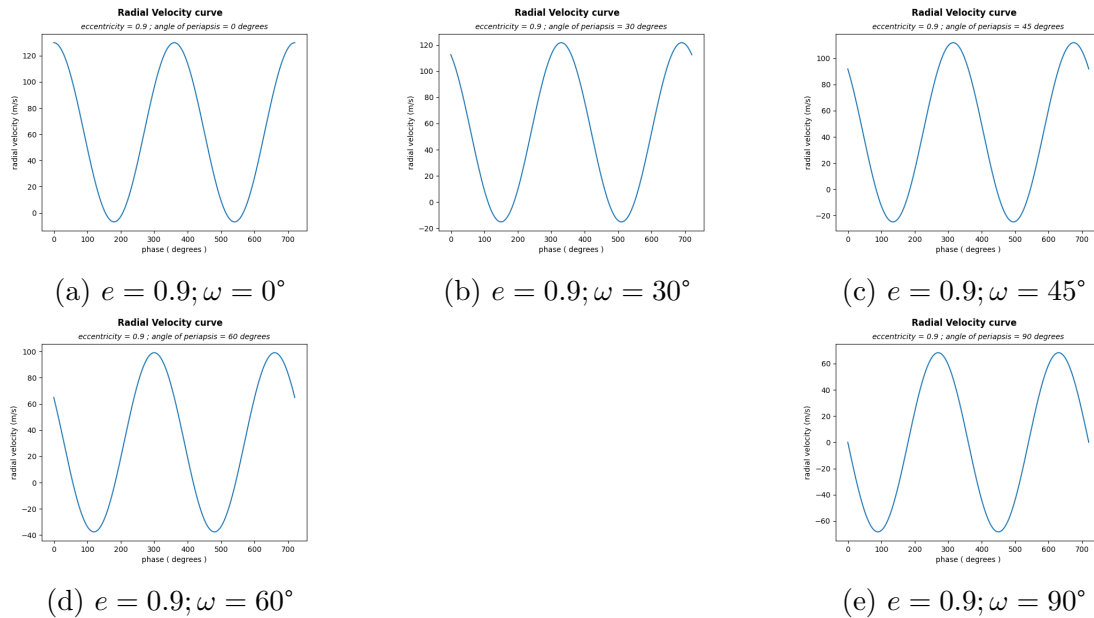
Figure C.3: Radial velocity curves w.r.t phase for different values of ω for eccentricity = 0.9

Appendix D

Radial Velocity v/s Time Graphs

The following graphs are modelled from the following parameter values: the mass of the star is given as 1 solar mass, orbital period is assigned to be 1 year, angle of inclination as 80°, semi-major axis of the star's orbit is given as 0.0005AU and the other parameters, eccentricity and ω are varied to show how a radial velocity curve varies when the ω value change for a given eccentricity and also when eccentricity value changes for a given ω value. The used eccentricity and ω values are:

a) When e = 0; ω = 20°

b) When e = 0; ω = 60°

c) When e = 0.4; ω = 20°

d) When e = 0.4; ω = 60°

e) When e = 0.9; ω = 20°

f) When e = 0.9; ω = 60°

It can be seen that for the same value of ω as the eccentricity value changes, the curve becomes less sinusoidal and starts to look more like a saw-tooth shape though not exactly a saw-tooth shape. The values start at the same point but the curve gets widened and narrowed at some regions. For same eccentricity cases where the periapsis angle value changes, the shape of the curve remains to be the same but there is a shift observed.
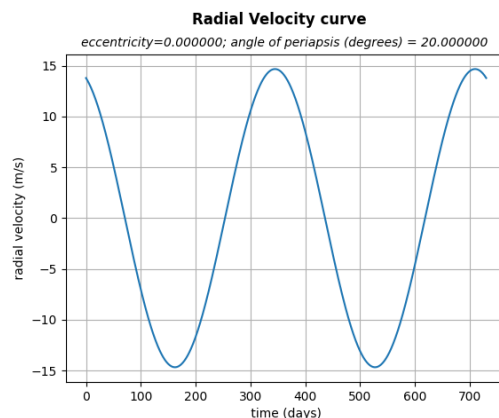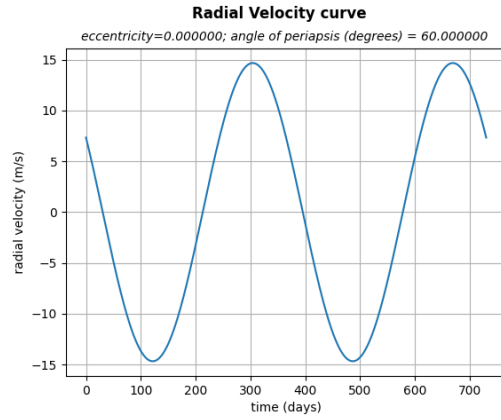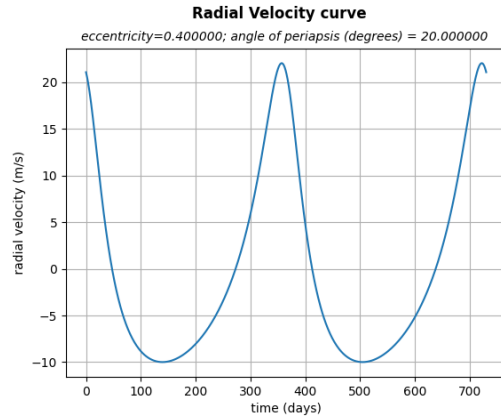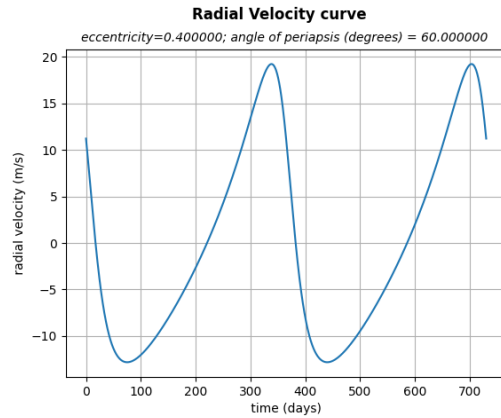

Figure D.1: Radial velocity curve with e = 0; ω=20°

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Figure D.2: Radial velocity curve with e = 0; ω=60°

Figure D.3: Radial velocity curve with e = 0.4; ω=20°

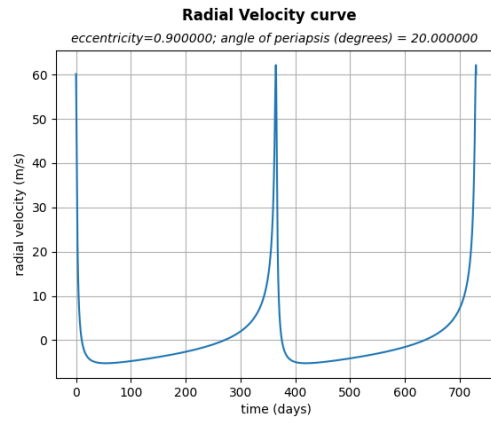

Figure D.4: Radial velocity curve with e = 0.4; ω=60°

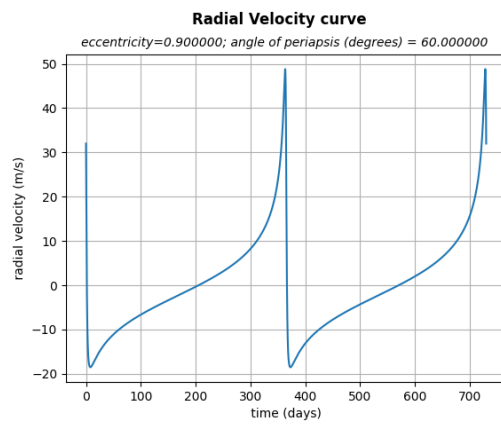Figure D.5: Radial velocity curve with e = 0.9; ω=20°

Figure D.6: Radial velocity curve with e = 0.9; ω=60°

Appendix E

Definitions

1 **Habitable Zone**: Also called as the Goldilocks zone or the circumstellar habitable zone, this is the region around a star in which, a planet can planet can support liquid water given that it has an atmospheric pressure accordingly.

1.1 **Exoplanets**: Planets that have been found beyond our solar system are called as the exoplanets, or the extra-solar planets. While most of the detected planets revolve around stars, some have been found to be on their own, meaning they are not orbiting a star in particular and are found to be moving around the galactic center. Such planets that are on their own are called as rogue planets.

1.1 **Pulsar Timing Method**: Pulsars, which are a type of neutron stars, emit radio waves at regular intervals of time as they rotate. Timing these pulses emitted by pulsars can help in understanding its orbit. This technique was not initially developed to detect exoplanets but this technique helps in detecting even planets of very small masses, which the other detection techniques fail to detect, from the orbital parameters that obtained from measuring the timings. As pulsars are less in number in comparison to the other stars, this technique cannot be widely used in detecting planets. Also, though the first observed exoplanet itself was around a pulsar PSR1257+12 and was indeed detected by this method, it is not easy for a planet to be formed around a pulsar and so such cases are not very often encountered. There are also other timing methods such as eclipsing binary times, pulsating hot subdwarfs etc. [45].

1.1.2 **Light Curve**: A light curve is a plot of the brightness of a celestial object over time. Light curve of a star is observed to find the variation in the brightness of the star in order to detect the presence of an exoplanet.

1.1.3**Einstein Ring**: Also known as Chwolson ring or the Einstein-Chwolson ring, is a ring that is produced when light from a distant celestial object encounters with some body on its way to the observer.

1.1.3**Rogue Planets**: Planets that are not bound to a solar system in particular and moves on its own through the galaxy.

1.1.5 **Barycentric motion**: Motion of an object around the barycenter,i.e., the center of mass.

2.1**Barycentric correction**: The frame of reference in which the observation is carried out may not be inertial in nature. So there is a need to transform from the

frame in which the observation is taking place to an inertial one and also to transform the frame of the source to inertial. This will allow both the emitting and receiving frames to be inertial in nature. In order to achieve this, a correction is done to the arrival time of the photons coming from the source. Our solar system is assumed to be almost inertial and the system's acceleration is taken to be almost negligible. So, the correction will place the hypothetical ideal receiver at the barycenter of the solar system. This correction is called as the barycentric correction.

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