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Reg. No......................................

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B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, FEBRUARY 2022

Second Semester

Complementary Course—INTEGRAL CALCULUS AND MATRICES

(Common for B.Sc. Physics, Chemistry, Petrochemicals, Geology, Food Science and Quality Control and Computer Maintenance and Electronics)

(2013 to 2016 Admissions)

Time : Three Hours
Maximum Marks : 80

Part A

Answer **all** *questions from this part.*
Each question carries 1 *mark.*

1. Find $\dfrac{dy}{dx}$ if $y = \int_{0}^{x} \sqrt{1+t^2}\ dt$.

2. State the mean value theorem for definite integrals.

3. Give an example of an odd function defined on the real line.

4. Write the formula for the length of a smooth curve $x = g(y), c \le y \le d$.

5. Write the surface area formula for revolution about the x-axis.

6. If $\int_{3}^{4} f(x)dx = 5$ and $\int_{4}^{8} f(x)dx = 8$, find $\int_{3}^{8} \dfrac{f(x)}{8}\ dx$.

7. Find $\int_{\pi/4}^{0} \sec x \tan x\ dx$.

8. State Cayley-Hamilton theorem.

9. What are the characteristic values of the zero matrix of order 3×3 ?

10. If a system of 'n' non-homogeneous equations in 'n' unknowns has a unique solution, then what is the rank of the coefficient matrix ?

$(10 \times 1 = 10)$

Turn over

Part B

Answer any **eight** *questions.*
Each question carries 2 marks.

11. Find the average value of $f(x) = 3x^2 - 3$ on $[0,1]$.

12. Suppose f is continuous and that $\int_0^3 f(z)dz = 3$ and $\int_0^4 f(z)dz = 7$. Find $\int_{40}^3 f(t)\, dt$.

13. Find the area of the region R enclosed by the parabola $y = x^2$ and the line $y = x + 2$.

14. Find the area bounded by the coordinate axes and the line $x + y = 1$.

15. Evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cot\theta\, \mathrm{cosec}^2\theta\, d\theta$.

16. Find the volume of the solid generated by revolving $y = x^3, y = 0, x = 2$ about the x-axis.

17. Evaluate $\int_{-1}^{1} \int_{1}^{2} x\, dx\, dy$.

18. Evaluate $\int_{1}^{4} \left(\frac{3}{2}\sqrt{x} - \frac{4}{x^3} \right) dx$.

19. Find the volume of the solid generated by revolving $y = x - x^2, y = 0$ about the x-axis.

20. Find the eigen values of the matrix $\begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix}$.

21. Find the characteristic polynomial of $A = \begin{bmatrix} 1 & 3 & 0 \\ -2 & 2 & -1 \\ 4 & 0 & 2 \end{bmatrix}$.

22. If A is an $n \times n$ non-singular matrix, show that $\left(A^\tau \right)^{-1} = \left(A^{-1} \right)^\tau$.

(8 × 2 = 16)

Part C

Answer any **six** *questions.*
Each question carries 4 marks.

23. Converting to polar integral, evaluate $\iint_R \dfrac{\ln\left(x^2 + y^2\right)}{\sqrt{x^2 + y^2}}\, dR,$ over the region $1 \le x^2 + y^2 \le e^2$.

24. Find the area of the regions enclosed by the curves $x^3 - y = 0$ and $3x^2 - y = 4$.

25. Find the length of the curve $y = x^{\frac{\pi}{2}}$ from $x = 0$ to $x = 4$.

26. The region in the first quadrant enclosed by the parabola $y = x^2$, the y-axis and the line $y = 1$ is revolved about the line $x = \dfrac{3}{2}$ to generate a solid. Find the volume of the solid.

27. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$.

28. Evaluate $\displaystyle\int_0^1 \int_0^2 \int_0^{1-x} dy\, dx\, dz.$

29. Find the rank of the matrix $A = \begin{bmatrix} 4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \end{bmatrix}$.

30. Find the eigen values and eigen vectors of $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.

31. Solve the following system of equations :

$$4x + y + z = 4$$
$$x + 4y - 2z = 4$$
$$3x + 2y - 4z = 6.$$

(6 × 4 = 24)

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (i) Evaluate $\int\limits_0^1\int\limits_0^1\int\limits_0^1 \left(x^2 + y^2 + z^2\right) dz\, dy\, dx.$

    (ii) Evaluate $\int\int xy(x+y)\, dxdy$, over the region between $y = x$ and $y = x^2$.

33. (i) Find the area of the region between the curve $y = 3 - x^2$ and the line $y = -1$.

    (ii) Find the volume of the region between the cylinder $z = y^2$ and the xy-plane that is bounded by the planes $x = 0, x = 1, y = -1, y = 1$.

34. (i) Find the area of the region common to the interiors of the cardioids $r = 1 + \cos\theta$ and $r = 1 - \cos\theta$.

    (ii) Find the length of the curve $y = \dfrac{4\sqrt{2}}{3}x^{\frac{3}{2}} - 1, 0 \le x \le 1.$

35. (i) Find the rank of the matrix $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$.

    (ii) Solve the following system of equations :

    $5x_1 + x_2 + x_3 + x_4 = 4$
    $x_1 + 7x_2 + x_3 + x_4 = 12$
    $x_1 + x_2 + 6x_3 + x_4 = -5$
    $x_1 + x_2 + x_3 + 4x_4 = -6.$

    $(2 \times 15 = 30)$