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Reg. No................................

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M.Sc. DEGREE (C.S.S.) EXAMINATION, NOVEMBER 2021

Fourth Semester

Faculty of Science

Branch I – (A)–Mathematics

MT 04 E14—CODING THEORY

(2012–2018 Admissions—Supplementary/Mercy Chance)

Time : Three Hours   Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. Find the dual code $C^{\perp}$ for the code $C=<S>$, where $S=\{0100, 0101\}$.

2. The generating matrix $G=\begin{bmatrix}1&0&0&1&1\\0&1&0&1&0\\0&0&1&0&1\end{bmatrix}$ for a code is given. Encode (i) $u = 100$; (ii) $v = 010$.

3. Show that if C is a self-dual code with generator matrix $(I\,|\,A)$ then C also has $(-A'\,|\,I)$ as generator matrix.

4. Define a perfect code. Give an example of a perfect code.

5. Show that the order of a subgroup H of G divides the order of G.

6. What is an irreducible polynomial. Test whether $f(x)=1+x+y^4$ is irreducible over $k=\{0,1\}$.

7. Prove that the integers are a PIR.

8. Define Vandermonde determinants.

(5 × 1 = 5)

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. Show that if d is the minimum weight of a code C, then C can correct $t=[(d-1)/2]$ or fewer errors and conversely.

10. Show that a self-dual $\left[n, \frac{n}{2}\right]$ ternary code exists iff n is divisible by 4.

Turn over

11. Show that if an $[n,k]$ code C has a generator matrix G = (I, A) in standard form, then a parity check matrix of C is $H=(-A^T, I)$, where A^T is the transpose of A and I is the $(n-k)\times(n-k)$ identity matrix.

12. Suppose $GF(2^3)$ was constructed using $1+x+x^3$. Suppose that $1+x+x^3+x^6$ is received. Find the code word send through the cyclic Hamming code of length 7.

13. Prove that if p is prime, then the integer mod p, GF(p) constitute a field. Also prove that every finite field F contains a subfield that is GF (p) upto relabelling for some prime p and $p.\alpha = 0$ for every $\alpha \in F$.

14. Show that $x^2 - 2$ is irreducible over GF (5).

15. Prove that the binary cyclic code with generator polynomial $1 + x$ is the $(n - 1)$ dimensional code C consisting of all even weight vectors of length n.

16. Prove that the value of the Vandermonde determinant D is $\prod_{j=1}^{n-1}\prod_{i=j+1}^{n}(\alpha_i - \alpha_j)$.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. (a) Prove that the covering radius has the following properties :

    (i) r is the weight of the coset of largest weight.

    (ii) r is the smallest among the number s so that every syndrome is a combination of s or fewer columns of any parity check matrix.

    (b) Prove that if $\subset$ is an $[n,k,d]$ code over GF (q), then $\left(\binom{n}{0}+(q-1)\binom{n}{1}+\ldots+(q-1)^t\binom{n}{t}\right)q^k \sum q^n$.

18. (a) Find a generator matrix for the linear code generated by :

    $S = \{11111111, 11110000, 11001100, 10101010\}$. Also give the parameters n, k, and d.

    (b) If d is even show that $A(n-1, d-1) = A(n,d)$.

19. Define Golay code. Decode each of the following received words using Golay codes :

(a) 111 000 000 000. (b) 011 011 011011.

20. (a) Using the double error correcting BCH code, decode the received vectors :

$x=(1,1,0,1,1,1,1,0,1,0,1,1,0,0,1)$, given the parity check matrix :

$$H=\begin{bmatrix} 1000 & 1000 \\ 0100 & 0001 \\ 0010 & 0011 \\ 0001 & 0101 \\ 1100 & 1111 \\ 0110 & 1000 \\ 0011 & 0001 \\ 1101 & 0011 \\ 1010 & 0101 \\ 0101 & 1111 \\ 1110 & 1000 \\ 0111 & 0001 \\ 1111 & 0011 \\ 1011 & 0101 \\ 1001 & 1111 \end{bmatrix}$$

(b) Messages are encoded using the above code. Determine if possible the locations of the errors is w is received and the syndrome WH is 0100 0101.

21. (a) Let C_1 and C_2 be cyclic codes with generator polynomial $q_1(x)$ and $q_2(x)$ and idempotent generators $e_1(x)$ and $e_2(x)$. Prove that $C_1 \cap C_2$ has a generator polynomial l.c.m. $\{q_1(x), q_2(x)\}$ and has idempotent generator $e_1(x), e_2(x)$ and $c_1 + c_2$ has a generator polynomial g.c.d. $\{q_1(x), q_2(x)$ and has idempotent generator $e_1(x)+e_2(x)-e_1(x)e_2(x)$.

(b) Prove that every minimal ideal μ is a field. If $\mu=\langle \hat{f}_i(x) \rangle$, show that μ can be constructed by using residues modulo the polynomial $f_i(x)$, where $f_i(x)$ satisfy $x^n-1=(x-1)f_1f_2\ldots f_k$, the factorization of x^n-1 into irreducible factors. Also prove that if $f_i(x)$ has degree m, then μ is isomorphic to the field $GF(2^m)$.

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22. (a) If C is an ideal in $R_n = F[x]/(x^n - 1)$ and let $g(x)$ be the monic polynomial of smallest degree in C ; prove that $g(x)$ is uniquely determined on $C = \langle g(x) \rangle$.

(b) Prove that of C is an ideal in R_n, the unique monic generator, $g(x)$ of C of smallest degree divides $x^n - 1$ and conversely if a polynomial $g(x)$ in C divides $x^n - 1$, then $g(x)$ has the lowest degree in $\langle g(x) \rangle$.

$(3 \times 5 = 15)$