21002071

M Sc DEGREE (CSS) EXAMINATION, NOVEMBER 2021

First Semester

CORE - PH010101 - MATHEMATICAL METHODS IN PHYSICS - I

M Sc PHYSICS,M.Sc.Space Science

2019 ADMISSION ONWARDS

D49C59D3

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Find div $\vec{F}$ and curl $\vec{F}$ where $\vec{F} = \mathrm{grad}(x^3 + y^3 + z^3 - 3xyz)$

2. State Stoke's Theorem.

3. Obtain the scale factors in cylindrical Co-ordinates.

4. Using inner product explain normalization and orthogonality of vectors.

5. Can basis be unique for a linear vector space? Explain.

6. Show that diagonal elements of a Hermitian matrix are real.

7. Show that the trace of a matrix remains invariant under similarity transformation.

8. Prove that the characteristic roots of a real orthogonal matrix are of modulus unity.

9. Contrast between dummy index and free index in mathematical notations.

10. Prove that the covariant derivative of g_{jk} is zero.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Express $\nabla \cdot \vec{A}$ in orthogonal curvilinear coordinates.

12. Express Laplacian operator in spherical polar and cylindrical co-ordinates.

13. A dice is rolled 9 times. Find the probabilities of having a 4 upwards (a) 3 times and (b) less than 4 times.

14. Summarize the essential features of normal distribution.

15. Obtain the direct sum of the matrices $A = [\,a\,]$, $B = \begin{bmatrix} b & c \\ d & e \end{bmatrix}$ and $C = \begin{bmatrix} f & g & h \\ i & j & k \\ x & y & z \end{bmatrix}$.

16. Find the inverse of the matrix $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ by Gauss- Jordan method.

17. If A_r^{pq} is a tensor, show that A_r^{pr} is a tensor of rank one.

18. Determine the Christoffel symbols of the first and second kind in elliptical cylindrical coordinates.

<div align="right">(6×2=12 weightage)</div>

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. Prove that a spherical coordinate system is orthogonal. Express the velocity and acceleration of a particle in spherical coordinates.

20. Prove the Cauchy-Schwarz inequality.

21. Solve the following equations by Gauss elimination method, $x + y + z + w = 0$.
$$x + y + z - w = 0$$
$$x + y - z + w = 0$$
$$x - y + z + w = 0$$

22. Find (a) g and (b) g^{jk} corresponding to $ds^2 = 5(dx^1)^2 + 3(dx^2)^2 + 4(dx^3)^2 - 6dx^1 dx^2 + 4dx^2 dx^3$.

<div align="right">(2×5=10 weightage)</div>