QP CODE: 21002033

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Reg No : ....

Name : ....

M Sc DEGREE (CSS) EXAMINATION, NOVEMBER 2021

First Semester

CORE - ME010101 - ABSTRACT ALGEBRA

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

22D13FAD

Time: 3 Hours | Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. *Check whether the group* $\mathbb{Z}_2 \times \mathbb{Z}_3$ *is cyclic.*
2. *Is every group a* G *- set? Justify your answer.*
3. *Let* G *be an abelian group. Show that the elements of finite order in* G *form a subgroup.*
4. Find the kernel of the homomorphism $\phi : \mathbb{Z}_{18} \to \mathbb{Z}_{12}$, where $\phi(1) = 10$.
5. Define (a) p-subgroup and (b) Sylow p-subgroup of a group G with examples.
6. Prove that every group of prime-power order is solvable.
7. *Compute the evaluation homomorphism* $\phi_2(x^2+3)$, $F = E = \mathbb{C}$
8. *Check* $25x^5 - 9x^4 - 3x^2 - 12$ *is irreducible over* $\mathbb{Q}$
9. Define ideal of a ring. Give an example.
10. Find atleast one $c \in \mathbb{Z}_5$ such that $\mathbb{Z}_5[x]/(x^2+cx+1)$ is a field.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. *Are the groups* $\mathbb{Z}_8 \times \mathbb{Z}_{10} \times \mathbb{Z}_{24}$ *and* $\mathbb{Z}_4 \times \mathbb{Z}_{12} \times \mathbb{Z}_{40}$ *isomorphic? Why or why not?*
12. *Show that the set of all* $g \in G$ *such that* $i_g : G \to G$ *is the identity inner automorphism* i_e *is a normal subgroup of a group* G.
13. Let G be a group of order p^n and let X be a finite G-set. Prove that $|X| \equiv |X_G| \pmod p$.

14. If p and q are distinct primes with p < q, then prove that every group G of order pq is not simple. Furthermore, if q is not congruent to 1 modulo p, then prove that G is abelian.

15. *Prove that the set G_n of non zero elements of $\mathbb{Z}_n$ that are not zero divisors forms a group under multiplication modulo n.*

16. *Let F be the desired field of quotients of an integral domain D. For [(a,b)] and [(c,d)] in F the addition is defined as [(a,b)] +[(c,d)] = [(ad+bc,bd)].Show that addition is commutative and associative.*

17. Show that if R, R' and R'' are rings and if $\phi : R \to R'$ and $\psi : R' \to R''$ are homomorphisms, then the composite function $\psi : R' \to R''$ is a homomorphism.

18. Explain prime ideal and maximal ideal of a ring and give examples.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. (a) Let X be a G – set. Prove that $\{g \in G/gx = x\}$ is a subgroup of G for each $x \in X$.
(b) Let X be a G – set and let $x \in X$. Prove that $|Gx| = (G:G_x)$. Also prove that if $|G|$ is finite, then $|Gx|$ is a divisor of $|G|$.

20. (a) State and prove third Sylow theorem.
(b) Prove that no group of order 42 is simple.

21. *(a) Explain the division algorithm in F[x].*
(b) If G is a finite subgroup of the multiplicative group of a field F, then prove that G is cyclic.

22. If G is any group and R is a commutative ring with nonzero unity, then show that group ring (RG,+, .) is a ring.

(2×5=10 weightage)