QP CODE: 22000351

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Reg No : ....

Name : ....

MSc DEGREE (CSS) EXAMINATION , JANUARY 2022

Second Semester

CORE - ME010202 - ADVANCED TOPOLOGY

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 Admission Onwards

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Time: 3 Hours | Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Show that a continuous bijection from a compact space onto a Hausdorff space is a homeomorphism
2. Show that every compact T_2 space is T_3
3. Define Cartesian product of the family of sets$\{ X_i/i \in I \}$
4. Distinguish between a cube and a Hilbert cube.
5. Given a product space which is connected. Prove that each coordinate space is connected
6. Define the term distinguish points. Give an example.
7. State a condition under which a topological space is embeddable in the Hilbert cube.
8. Define a sequentially compact space. Give an example to show that compactness does not imply sequential compactness.
9. Define cluster point of a net.
10. If a net converges to a point, prove that so does every subnet of it.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. By stating Urysohn lemma Show that All T_4 spaces are Tychonoff spaces

12. Prove that

a) Not all continuous functions can be continuously extended.

b) If A is a subset of a space X and f is a continuous real valued function on A, then any two extensions of f agrees on the closure of A.

13. For any sets Y, I and J, prove that $(Y^I)^J = Y^{I \times J}$ upto a set theoritic equivalence.

14. Define productive property . Give an example of a productive property.

15. Explain evaluation function. Characterise evaluation function of a family of functions.

16. Prove that a metric space is compact iff it is countably compact.

17. Show that if each net in a topological space converges to a unique point , then the space is Hausdorff.

18. Define products of two paths.Show that the product operation on paths induces a well defined operation on path homotopy classes.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. Show that any continuous real valued function on a closed subset of a normal space can be continuously extended to the whole space

20. (a) Let C_i be a closed subset of a space X_i for $i \in I$. Prove that $\prod_{i \in I} C_i$ is a closed subset of $\prod_{i \in I} X_i$ with respect to product topology.

(b) If X_i is a T_1 space for each $i \in I$. Prove that $\prod_{i \in I} X_i$ is a T_1 space in the product topology

21.

(a)Let $\{f_i : X \to Y_i | i \in I\}$ be af amily of continuous functions which distinguishes points and also distinguishes point from closed sets. Then Prove that the corresponding evaluation map is an embedding of X into the product space $\prod_{i \in I} Y_i$.

(b)Prove that a topological space is completely regular if and only if the family of all continuous real valued functions on it distinguishes points from closed sets.

22. When you say that a net converges.Let X be a topoplogical space with a sub-base $\mathcal{S}$ and x in X. Prove that a net in X converges to x iff the condition in the definition of converges holds for all neighbourhoods of x which are members of $\mathcal{S}$

(2×5=10 weightage)