MSc DEGREE (CSS) EXAMINATION , JANUARY 2022

Second Semester

CORE - PH010201 - MATHEMATICAL METHODS IN PHYSICS-II

M Sc PHYSICS,M.Sc.SPACE SCIENCE

2019 Admission Onwards

421C74D1

Time: 3 Hours   Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Find the analytic function $f(z) = u(x,y) + iv(x,y)$ in which $v(x,y) = e^{-y}\sin x$.

2. Evaluate using Cauchy's integral formula, $\oint_c \frac{z^3}{(z-1)^3}\, dz$, where c is the circle $|z| = 3$.

3. Classify the singularities and calculate the reside for $f(z) = \frac{sin z}{z^4}$.

4. Give the Fourier integral representation for an odd function.

5. Discuss any two properties of Inverse Fourier transform?

6. Obtain the Laplace transform of a periodic function with period T .

7. Evaluate $\int_0^\infty e^{-x^4}\, dx$.

8. If $H_n(x)$ represents Hermite polynomial of degree n ,show that $H_{2n}(0) = (-1)^n \frac{(2n)!}{n!}$.

9. Write down the three dimensional Laplace equation .Mention any two areas of its application.

10. Write down the Helmholtz equation in three different coordinate systems.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Verify Cauchy's theorem for the integral of z^2 taken over the boundary of the rectangle with vertices $-1 - i,\ 1 - i,\ 1 + i,\ -1 + i$.

12. Evaluate $\int_0^{2\pi} \frac{d\theta}{5 + 4\cos\theta}$ using contour integration

13. Expand the function $f(x) = x^2$, $0 < x < 2\pi$ as a series of sines and cosines.

14. Give the momentum representation of hydrogen atom using Fourier transform.

15. Prove that Beta function $\beta(a, b)$ is symmetric in its arguments.

16. If $J_n(x)$ is n^{th} order Bessel function, show that $\frac{d}{dx}[x^{-n} J_n(x)] = -x^{-n} J_{n+1}(x)$.

17. If $P_n(x)$ is Legendre polynomial of degree n , show that $x\, P_n'(x) - P_{n-1}'(x) = n\, P_n(x)$.

18. Explain the method of Green's function for solving nonhomogeneous differential equations.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. a) Obtain Laurent's series expansion of a complex function. b) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ as a Laurent series valid for $(a)\ 1 < |z| < 3$ $(b)\ 0 < |z + 1| < 2$ $(c)\ |z| > 3$

20. Derive Lapace transform of nth order derivative of a function. Also solve for damped harmonic oscillator using Laplace transform.

21. Discuss Rodrigues Formula, Orthogonality relation and one recurrence relation of Associated Legendre Polynomial.

22. Solve the Heat conduction equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{2}\frac{\partial u}{\partial t}$ over $0 < x < 3, t > 0$ for the boundary conditions $u(0, t) = u(3, t) = 0$ and the initial condition $u(x, 0) = 5\sin(4\pi x)$

(2×5=10 weightage)