QP CODE: 21002036

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Reg No : ....

Name : ....

M Sc DEGREE (CSS) EXAMINATION, NOVEMBER 2021

First Semester

CORE - ME010104 - REAL ANALYSIS

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

94FDAC0B

Time: 3 Hours | Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Let f be continuous on $[a, b]$. Then prove that f is of bounded variation on $[a, b]$ only if f can be expressed as the difference of two increasing continuous functions.
2. Let f and g be complex valued functions defined as follows : $f(t) = e^{2\pi it}$ if $t \in [0, 1]$ and $g(t) = e^{4\pi it}$ if $t \in [0, 1]$. Then prove that the length of g is twice that of f.
3. *Define upper and lower Riemann Stieltjes sum for a real bounded function.*
4. *Show that* $\underline{\int_a^b} f d\alpha \leq \overline{\int_a^b} f d\alpha$.
5. *If* $f \in \mathscr{R}(\alpha)$ *on [a, b] then show that* $|f| \in \mathscr{R}(\alpha)$.
6. Define pointwise convergence of sequence of functions.
7. When a series $\Sigma f_n(x)$ is said to converge uniformly on a set ?
8. Every convergent sequence is a Cauchy sequence. What about the converse?
9. If K is compact, if $f_n \in \mathscr{C}(K)$ for $n = 1, 2, 3, \ldots,$ and if $\{f_n\}$ is pointwise bounded and equicontinuous on K, then prove that $\{f_n\}$ is uniformly bounded on K.
10. Prove that $\lim\limits_{x \to 0} \frac{log(1+x)}{x} = 1$

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Show that $f(x) = x^2 \cos \dfrac{1}{x}$ if $x \neq 0$ and $f(0) = 0$ is of bounded variation on $[0, 1]$.

12. Let f be of bounded variation on $[a,b]$. Let V be defined on $[a,b]$ as follows: $V(x) = V_f(a,x)$ if $a < x \leq b$, $V(a) = 0$. Then prove that
(i). V is an increasing function on $[a,b]$.
(ii). $V - f$ is an increasing function on $[a,b]$.

13. If a < s < b , f is bounded on [a,b] , f is continuous at ***s*** and α(x) = *I*(x-***s***), where *I* is the unit step function, then prove that $\int_a^b f d\alpha = f(s)$.

14. *If $f \in \mathscr{R}$ on [a,b] and if there is a differential function F on [a,b] such that F′ = f , then prove that* $\int_a^b f(x)dx = F(b) - F(a)$.

15. Prove that the function $f_n(x) = n^2x(1-x^2)^n\, ; 0 \leq x \leq 1$, converges to a continuous function although the convergence is not uniform.

16. Obtain a series from $\phi(x) = |x|, (-1 \leq x \leq 1)$ and $\phi(x+2) = \phi(x)$ for all real x , which converges uniformly on R^1.

17. Suppose $\{f_n\}$ is an equicontinuous seqeuce of functions on a compact set K, and $\{f_n\}$ converges pointwise on K. Prove that $\{f_n\}$ converges uniformly on K.

18. For the double sequence $a_{ij}, i = 1,2,3,..,j = 1,2,3,\dots,$ suppose that $\sum_{j=1}^{\infty} |a_{ij}| = b_i,\ (i = 1,2,3,\dots)$ and $\sum b_i$ converges. Prove that $\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} a_{ij} = \sum_{j=1}^{\infty}\sum_{i=1}^{\infty} a_{ij}$.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. (i) State and prove additive property of arc length function $\Lambda_f(x,y)$ for a rectifiable curve *f*.
(ii) Define $s(x) = \Lambda_f(a,x)$ for $x \in [a,b]$ and let $s(a) = 0$ for a rectifiable path f defined on $[a,b]$. Then prove that the function f is increasing and continuous on $[a,b]$.
(iii) Let $f : [a,b] \to R^n$ and $g : [c,d] \to R^n$ be two paths in R^n, each of which is one to one on its domain. Then prove that $\mathbf{f}$ and $\mathbf{g}$ are equivalent if and only if they have the same graph.

20. *(i) If f is continuous on [a, b] then show that f ∈ $\mathscr{R}$ (α).*
*(ii) If f is monotonic on [a, b] and if **α** is continuous on [a, b] then prove that f ∈$\mathscr{R}$(α).*

21. Let α be monotonically increasing on $[a,b]$. Suppose $f_n \in \mathscr{R}(\alpha)$on $[a,b]$, for $n = 1,2,3,\dots$ and suppose $f_n \to f$ uniformly on $[a,b]$. Then prove that $f \in \mathscr{R}(\alpha)$ on $[a,b]$ and $\int_a^b f d\alpha = \lim_{n\to\infty} \int_a^b f_n d\alpha$.
Also show that if the series $f(x) = \sum_{n=1}^{\infty} f_n(x), (a \leq x \leq b)$ converges uniformly on $[a,b]$, then $\int_a^b f\, d\alpha = \sum_{n=1}^{\infty} \int_a^b f_n\, d\alpha$.

22. State and prove Weierstrass approximation theorem.

(2×5=10 weightage)