QP CODE: 21102025

21102025

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS)EXAMINATION, AUGUST 2021

Third Semester

Core Course - MM3CRT01 - CALCULUS

Common to B.Sc Computer Applications Model III Triple Main, B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

B52A177D

Time: 3 Hours Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Find the points of inflection of the curve y = x^3 - $3x^2$ - 9x +9.
2. Write the formula for radius of curvature in cartesian co-ordinates.
3. Find the centre of curvature at the given point on the curve xy = c^2 ;(c,c)
4. Find the envelope of the family of the semi-cubical parabola y^2 = $(x+ a)^2$.
5. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ if $f(x, y) = x^2 - y^2$
6. Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if
   $\text{if}\ w = x^2 - y^2,\ x = r - s,\ y = r + s$
7. Define saddle point of a two variable function $f(x, y)$ at a critical point (a, b)
8. How to obtain the volume of solid of revolution generated by rotating the region between the X-axis and graph of the function $y = f(x)$; $a \leq x \leq b$ about X-axis.
9. If $R(x)$ and $r(x)$ denote the outer and inner radius of cross section of a solid of revolution about X-axis, with hole at x ; $a \leq x \leq b$. Find the volume of solid.
10. Find the length of the curve $y = 2x - 1$ from $x = 1$ $\text{to}\ x = 2.$
11. Express the rectangular coordinates (x, y, z) in terms of spherical coordinate (ρ, ϕ, θ).
12. Define the Jacobian $J(u, v)$ of the co-ordinate transformation $x = g(u, v),\ y = h(u, v).$

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Using Maclaurin's series, prove that $e^x \sin x = x + x^2 + \frac{2}{3!}x^3 - \frac{2^2}{5!}x^5 + \ldots\ldots\ldots\ldots\ldots\ldots + \sin\left(\frac{n\pi}{4}\right) \frac{2^{n/2}}{n!}x^n + \ldots\ldots\ldots\ldots\ldots$.

14. Expand $\log(x+a)$ in powers of x , using Taylor's series.

15. Find the points closest to the origin on the hyperbolic cylinder $x^2 - z^2 - 1 = 0$

16. Find the greatest and smallest values that the function $f(x,y) = xy$ takes on the ellipse $x^2 + 4y^2 = 8$

17. The solid lies between planes perpendicular to the X-axis at $x = -1$ and $x = 1$ and the cross sections perpendicular to the X-axis are squares with side run from the semicircle $y = -\sqrt{1-x^2}$ to the semicircle $y = \sqrt{1-x^2}$. Find the area of cross section $A(x)$ and hence evaluate the volume of the solid.

18. Find the area of the surface that is generated by revolving the portion of the curve $x = \sqrt[3]{y}\,;\ 1 \le y \le 8$ about the X-axis.

19. Sketch the region of integration for $\int\int_R f(x,y)\,dA$ where where R is the region in the first quadrant of XY-plane bounded by the circle $x^2 + y^2 = 1 \text{ and the line } x + y = 1$. Write both equivalent integrals with order of integration reversed.

20. Sketch the region bounded by the coordinate axes and the line $x + y = 2$. Then express the region's area as double integral and evaluate the integral.

21. Write any four different triple integrals for the volume of the rectangular solid in the first octant bounded by the coordinate planes and the planes $x = 1, y = 2 \text{ and } z = 3$. Evaluate one of the integrals.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. a) Find the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$

b) Find the envelope of the line $\frac{x}{a} + \frac{y}{b} = 1$ where the parameters a and b are connected by the relation $a^2 + b^2 = c^2$.

23. (a). If $u = \tan^{-1}\left(\dfrac{x^3 + y^3}{x + y}\right)$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \sin 2u$

(b). Calculate $f_x, f_y, f_z, f_{xy}, f_{xz}$ if $f = e^{-(x^2+y^2+z^2)}$

(c). Find the point $P(x, y, z)$ closest to the origin on the plane $2x - y + 2z = 16$

24. (a).The region enclosed by the X-axis and the parabola $y = 2x - x^2$ is revolved about the vertical line $x = -1$ to generate a solid. Find the volume of the solid using shell method.

(b). Find the length of the curve $y = \dfrac{x^3}{12} + \dfrac{1}{x}$ from $x = 0$ to $x = 4$.

25. (a). Evaluate $\displaystyle\int_0^1 \int_0^{1-x^2} \int_3^{(4-x^2-y)} x\,dz\,dy\,dx$

(b). Evaluate the cylindrical coordinate integral $\displaystyle\int_0^{2\pi} \int_0^1 \int_r^{\sqrt{2-r^2}} dz\,r\,dr\,d\theta$

(2×15=30)