BHARATAMATA COLLEGE, THRIKKAKARA
MODEL EXAMINATION FEB.2020
B.Sc. DEGREE PROGRAMME- SEMESTER VI
MATHEMATICS (Core Course)
MM6CRT09-Real Analysis **Code: 6MM1**

Time: 3 Hrs. **Max.Marks:80**

Part A (*Answer any 10 questions. Each Question carries 2 marks)*

1. Define the uniform continuity of a function.
2. Define the Lipschitz function on a set A.
3. Define a monotone function.
4. Define jump of a function f at a point a.
5. State the chain rule for the differentiable functions.
6. State the Rolle's theorem.
7. Define the Signum function.
8. Define the primitive of a function f on [a, b].
9. Show that every constant function is Riemann integrable.
10. Show that Lim(x/n)=0 for all x in R.
11. State Bounded Convergence Theorem on Riemann Integrable.
12. State the Squeeze theorem.

Part B (*Answer any 6 questions. Each question carries 5 marks.)*

13. State and prove Uniform continuity theorem.
14. Let $I \subseteq R$ and let $f: I \to R$ be increasing on I. If $c \in I$, then f is continuous at c if and only if $j_f(c) = 0$
15. State and prove the Mean value theorem.
16. Prove that if $f: A \to R$ is a Lipschitz function , then f is uniformly continuous on A. Is the converse true? Justify your answer.
17. Let $I \subseteq R$ and let $f: I \to R$ be monotone on I. Then the set of points $D \subseteq I$ at which f is discontinuous is a countable set.
18. If F defined by F(z)=$\int_a^z f$ for $z \in [a, b]$ is continuous on $[a, b]$ and if

$|f(x)| \leq M \quad \forall x \in [a, b]$, then prove that $|F(z) - F(w)| \leq M|z - w| \ \forall\, z, w \in [a, b]$.

19. If $f: [a, b] \to R$ is is continuous on [a, b] then prove that $f \in R[a, b]$.

20. Find the integral $\int_1^4 \frac{\sin\sqrt{t}}{\sqrt{t}}\, dt$ by using substitution theorem.

Part C (*Answer any 2 questions. Each question carries 15 marks.)*

21. (a.) State and prove Continuous Extension theorem.

    (b) State and prove Continuous Inverse theorem.

22. (a) State and prove Caratheodory's theorem.

    (b) State and prove Darboux's theorem for differentiable function.

23. State and prove the Fundamental Theorem of Calculus (First & II form).

24. Let $f: [a, b] \to R$ and let $c \in (a, b)$ then prove that $f \in R[a, b]$ if and only if its restrictions

    to $[a, c]$ and $[c, b]$ are both Riemann integrable and $\int_a^b f = \int_a^c f + \int_c^b f$.

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