QP CODE: 21000689

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Reg No : ....

Name : ....

M Sc DEGREE (CSS) EXAMINATION, JULY 2021

Fourth Semester

Faculty of Science

Elective - ME800402 - ALGORITHMIC GRAPH THEORY

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 Admission Onwards

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Time: 3 Hours | Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Construct a graph of order 5 whose vertices have degrees $1, 2, 2, 3, 4$. What is the size of this graph?
2. Write an algorithm to determine the first word alphabetically from a list of n words, and output this word and its location in the list.
3. What is adjacency matrix of a graph? Draw the graph G be with vertex set $V(G) = \{v_1, v_2, v_3, v_4, v_5, v_6\}$, $E(D) = \{v_1v_2, v_1v_3, v_2v_3, v_3v_4, v_3v_5\}$. Find adjacency matrix of G.
4. Define a forest. Give an example.
5. State Cayley's Tree formula.
6. Define distance function on a graph G. Show that it is a metric.
7. Define vertex connectivity of a graph. Find $\kappa(K_{m,n})$
8. Define an edge disjoint $u - v$ path in a graph G and the term $\lambda(u, v)$, where $u, v \in V(G)$
9. *Define a feasible vertex labeling of a weighted complete bipartite graph*
10. *Define a* $\{b, v. r, k, \lambda\}$ *design and state Fisher's inequality*

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Define (a) a non-separable graph, (b) a block, (c) an end-block in a graph. Give examples for each.

12. (a) Explain indegree, outdegree and degree of a vertex in a digraph. Draw a digraph and find indegree, outdegree and degree of each vertex.
    (b) State and prove The First Theorem on Digraph Theorey.

13. If T is a balanced complete binary tree of height h and order p, then prove that $h = \lceil log_2(\frac{p+1}{2})\rceil$

14. Explain BFS Algorithm

15. Define a flow in a network N. Give an example of a flow where flow along each arc is a positive integer.

16. In a network, show that the value of a maximum flow equals the capacity of a minimum cut.

17. *Let G be a bipartite graph with partite sets V_1 and V_2. Prove that the set V_1 can be matched to a subset of V_2 if and only if V_1 is non deficient*

18. *Prove that every bridgeless cubic graph contains a 1-factor*

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. a) An edge e of a connected graph is a bridge if and only if e does not lie on any of the cycle on G.
    b) Show that every $u - v$ walk in a graph contains a $u - v$ path.

20. Write an algorithm to determine a critical path in an activity digraph D with start vertex S and terminal vertex T.

21. State and prove a necessary and sufficient condition that a flow f in a network N with underlying digraph D is a maximum flow.

22. *State and prove Berge's theorem to determine the maximum matching in a graph G.*

(2×5=10 weightage)