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Reg. No................................

Name...................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, FEBRUARY 2021

Third Semester

Faculty of Science

Branch I (A)—Mathematics

MT03C11—MULTIVARIATE CALCULUS AND INTEGRAL TRANSFORMS

(2012 – 2018 Admissions)

Time : Three Hours | Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. Define any four integral transforms.
2. Use convolution theorem to find a relationship between Beta and Gamma functions.
3. Define directional derivative with an example.
4. Obtain the matrix form of the chain rule.
5. Give example to show that two mixed partials $D_{1,2} f$ and $D_{2,1} f$ need not be equal.
6. State the theorem relating the Jacobian determinant of a complex valued function with its derivative.
7. Explain flip with an example.
8. Show that every k-form can be represented in terms of basic k-forms.

(5 × 1 = 5)

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. Obtain the exponential form of the Fourier Integral Theorem.
10. Derive the different forms of Fourier series.
11. Show that if the total derivative exists, it must be unique and the total derivative of a linear function is itself.
12. Establish the chain rule on the differentiability of composite functions.

Turn over

13. Use Taylor's formula to express :

    $f(x, y) = x^2 + xy + y^2$ in powers of $x - 1$ and $y - 2$.

14. Obtain the relation between the Jacobian determinant of a complex valued function with its derivative.

15. If σ is an oriented rectilinear k-simplex in an open set $E \subset R^n$ and if $\bar{\sigma} = \in \sigma$. Prove $\int_{\bar{\sigma}} w = \in \int_{\sigma} w$ for every k-form w in E.

16. (a) State Stokes theorem.

    (b) Define Lebesgue integral.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each has weight 5.

17. Suppose T is a $\mathscr{C}'$ – mapping of an open set $E \subset R^n$ into an open set $V \subset R^m$, ϕ is a k-surface in E, and w is a k-form in V prove $\int_{T\phi} w = \int_{\phi} w_T$.

18. State and prove Fourier Integral Theorem.

19. Explain the notions of partial derivative and total derivative. Also prove the result on the total derivative expressed in terms of partial derivatives.

20. Explain how matrices arise in connection with total derivatives.

21. State and prove that theorem to show that continuity of all but one of the partials imply differentiability.

22. Establish the second derivative test for extrema.

(3 × 5 = 15)