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Reg. No.....

Name.....

M.Sc. DEGREE (C.S.S.) EXAMINATION, MAY 2020

Fourth Semester

Faculty of Science

Branch I (A) : Mathematics

MT 04 E13—ALGORITHMIC GRAPH THEORY

(2012 Admission onwards)

Time : Three Hours Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. Define graph isomorphism. Give an example.
2. State Knapsack problem.
3. State Cayley's Tree formula.
4. Define distance function on a graph G and prove that it is a metric.
5. Show that if G is an n-edge-connected graph, then $G + K_1$ is $(n + 1)$-edge-connected.
6. Define edge-connectivity of a graph. Find $\lambda(K_{m,n})$.
7. What you meant by a feasible vertex labeling ?
8. Define a block design.

$(5 \times 1 = 5)$

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. Prove that every u-v walk contains a u-v path.
10. Write an algorithm to alphabetize a list $w(1), w(2), ...,w(n)$ of n words.
11. Prove that a tree of order p has size $p - 1$.
12. Prove that the center of every tree is isomorphic to K_1 or K_2.
13. Prove that $\lambda(G) \leq \delta(G)$. Give a sufficient condition for the equality holds.

Turn over

14. Define a flow in a network N. Give an example of a flow where the flow along each arc is a positive integer.

15. Prove that every r-regular bipartite multigraph $(r \geq 1)$ has a perfect matching.

16. Write the Kuhn-Munkres algorithm to find a maximum weight perfect matching in a weighted complete bipartite graph G.

$(5 \times 2 = 10)$

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. Prove that a nontrivial graph G is bipartite if and only if G has no odd cycle.

18. Explain binary search algorithm. What is its complexity ?

19. Explain critical path algorithm. Find its complexity ?

20. State and prove Whitney's theorem on n-connected graphs.

21. (a) Prove that a matching M in a graph G is a maximum matching if and only if there exists no augmenting path, with respect to M, in G.

    (b) Prove that every r-regular bipartite multigraph $(r \geq 1)$ has a perfect matching.

22. Explain a maximum matching algorithm for bipartite graphs and find its complexity.

$(3 \times 5 = 15)$