

**BHARATA MATA COLLEGE, THRIKKAKARA**  
**FIRST INTERNAL EXAMINATION JANUARY 2020**  
**M.Sc. MATHEMATICS SEMESTER IV**  
**COMBINATORICS**

TIME: 1½Hrs

Max. Weight: 15

**Section A**

**Answer any FOUR Questions, Each question carries 1weight**

1. State the Addition Principle and Multiplication Principle with one example each.
2. Find the number of positive divisors of 600 inclusive of 1 and 600 itself.
3. In how many ways can 5 boys and 3 girls be seated around a table if boy B1 and girl G1 are not adjacent.
4. State the Injection and Bijection Principle
5. Show that  $(4n)!$  is a multiple of  $2^{3n} \cdot 3^n$  for each natural number  $n$ .

**Section B**

**Answer any THREE Questions, Each question carries 2weight**

6. Let  $X = \{1, 2, \dots, 100\}$  and let  $S = \{(a, b, c) : a, b, c \in X, a < b, a < c\}$ . Find  $|S|$ .
7. Show that the number of positive divisors of " $\underbrace{111\dots1}_{1992}$ " is even.
8. Let  $M = \{\infty.a_1, \infty.a_2, \dots, \infty.a_n\}$ . Show that the number of  $r$ -element multisubsets of  $M$  is given by  $H_r^n = \binom{r+n-1}{r}$ .
9. If  $|X| = n$  show that  $|P(X)| = 2^n$  for all  $n$ .

**Section C**

**Answer any ONE Question, Each question carries 5weight**

10. (a) Define the Stirling number of the first kind  $s(r, n)$  and prove that for  $r, n \in \mathbb{N}$  with  $n \leq r$ ,  $s(r, n) = s(r-1, n-1) + (r-1)s(r-1, n)$ .  
(b) Find the number of nonnegative integer solutions to the equation  $x_1 + x_2 + \dots + x_n = r$ .
11. Let  $n$  and  $k$  be positive integers and let  $S$  be a set of  $n$  points in the plane such that  
(a) no three points of  $S$  are collinear  
(b) for any point  $P$  of  $S$ , there are at least  $k$  points of  $S$  equidistant from  $P$ .

Prove that  $k < \frac{1}{2} + \sqrt{2n}$ .

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