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Reg. No.................................

Name....................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, MAY 2020

Fourth Semester

Faculty of Science

Branch I-(A)—Mathematics

MT04 E01—ANALYTIC NUMBER THEORY

(2012 Admission onwards)

Time : Three Hours | Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. Prove that each function μ_k is multiplicative.

2. Prove $[x + n] = [x] + n$.

3. Determine all positive integers n such that $\left[\sqrt{n}\right]$ divides n.

4. Prove that there do not exist polynomials P and Q such that

$$\Pi(x) = \frac{P(x)}{Q(x)} \text{ for } x = 1, 2, 3, \ldots$$

5. If $a > 0$ and $b > 0$ show that $\dfrac{\pi(ax)}{\pi(bx)} \sim \dfrac{a}{b}$ as $x \to \infty$.

6. Prove the converse of Wilson's theorem.

7. Prove that no prime $p \equiv 3 \bmod 4$ is the sum of two squares.

8. Prove that m is prime if and only if $\exp_m(a) = m - 1$ for some a.

(5 × 1 = 5)

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. Prove that $\varphi(n) > \dfrac{n}{6}$ for all n with atmost 8 distinct prime factors.

Turn over

10. For p prime, show that $\mu(p, \cdot)$ is multiplicative.

11. Prove $\sum_{n \le k} \left[\sqrt{\frac{x}{n}}\right] = \sum_{n \le \sqrt{x}} \left[\frac{x}{n^2}\right]$.

12. Show that Selberg's formula implies $r + \delta = 2$.

13. Find all positive integers n for which $n^{17} \equiv n \bmod 408°$.

14. Let S be a set of n integers not necessarily distinct. Prove that some non-empty subject of S has a sum which is divisible by 7.

15. State and prove Lagrange's theorem.

16. If $| x | < 1$ prove that $\int_{m=1}^{\infty} (1 + x^m) = \int_{m=1}^{\infty} (1 - x^{2m-1})^{-1}$.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. Explain :
    - (i) Dirichlet inverse.
    - (ii) Dirichlet product of arithmetic function.
    - (iii) Asymptotic equality of function.
    - (iv) Average order of the divisor function.
    - (v) Generalised convolutions.

18. Define a completely multiplicative function and characterise it.

19. State and prove Shapiros Tauberian theorem.

20. State Chinese remainder theorem. Use it to state and prove the theorem concerning the set of lattice points visible from the origin.

21. (a) If a prime p does not divide a prove $a^{P-1} \equiv 1 \pmod{p}$.

    (b) For any prime p, prove that all the coefficients of the polynomial

    $$f(x) = (x - 1)(x - 2) \ldots\ldots (x - p + 1) - x^{P-1} + 1$$

    are divisible by p.

22. (a) Explain the geometric representation of partitions.

    (b) Establish the existence of primitive roots mod p^{α}.

(3 × 5 = 15)