

**BHARATA MATA COLLEGE, THRIKKAKARA**  
**FIRST INTERNAL EXAMINATION, JANUARY 2020**  
**M.Sc. MATHEMATICS, Semester IV**  
**ANALYTIC NUMBER THEORY**

TIME: 1<sup>1</sup>/<sub>2</sub> Hrs

Max. Weight: 15

**Section A**

**Answer any 4 Questions**  
**Each question carries 1 weight**

1. Prove that Dirichlet multiplication is commutative.
2. Prove that  $\varphi(mn) = \varphi(m)\varphi(n)$  if  $(m, n) = 1$ .
3. If  $f$  is multiplicative, prove that  $f(1) = 1$ .
4. Find a formula for the Bell series of Mobius function  $\mu$ .
5. Define divisor function  $\sigma_\alpha(n)$ . Interpret  $\sigma_0(n)$  and  $\sigma_1(n)$  in terms of divisors of  $n$ .

(4x1=4)

**Section B**

**Answer any 3 Questions**  
**Each question carries 2 weight**

6. Prove that the Mobius function  $\mu(n)$  is multiplicative.
7. For arithmetical functions  $\alpha$  and  $\beta$ , prove that  $\alpha \circ (\beta \circ F) = (\alpha * \beta) \circ F$ .
8. If  $\alpha$  has a Dirichlet inverse  $\alpha^{-1}$ , then prove that  $G(x) = \sum_{n \leq x} \alpha(n) F\left(\frac{x}{n}\right)$  if and only if  $F(x) = \sum_{n \leq x} \alpha^{-1}(n) G\left(\frac{x}{n}\right)$ .
9. Prove that the Mangoldt function  $\Lambda(n) = \sum_{d|n} \mu(d) \log\left(\frac{n}{d}\right) = - \sum_{d|n} \mu(d) \log d$ .

(3x2=6)

**Section C**

**Answer any 1 Question**  
**Each question carries 5 weight**

10. If  $f$  is multiplicative, then prove that  $f$  is completely multiplicative if and only if  $f^{-1}(n) = \mu(n)f(n)$ .
11. State and prove Euler's summation formula.

(5x1=5)