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Reg. No................................

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M.Sc. DEGREE (C.S.S.) EXAMINATION, NOVEMBER 2020

Second Semester

Faculty of Science

Branch I (a)—Mathematics

MT 02 C10—REAL ANALYSIS

(2012–2018 Admissions)

Time : Three Hours | Maximum Weight : 30

Part A

Answer any **five** *questions.*

Each question has weight 1.

1. Explain the terms bounded variation and total variation of a function *f* defined on $[a,b]$.

2. Is the function $f(x) = x^2 \sin\left(\frac{1}{x}\right)$ if $x \neq 0, f(0) = 0$ is of bounded variation. Justify your answer.

3. Define Riemann-Stieltjes integral of a function *f* with respect to α over $[a,b]$.

4. Show that if $f, g \in \mathrm{R}(\alpha)$ on $[a,b]$, then $f\ g \in \mathrm{R}(\alpha)$ on $[a,b]$.

5. Suppose $\{f_n\}$ is a sequence of functions defined on E and suppose $|f_n(x)| \leq \mu_n, x \in \mathrm{E}$ and $n = 1, 2, \ldots,$ show that $\sum f_n$ converges uniformly on E if $\sum \mu_n$ converges.

6. Consider $f(x) = \sum_{n=1}^{\infty} \frac{1}{1+n^2 x}$. On what interval does the series converge uniformly.

Turn over

7. Define Fourier series of an integrable function on $[-\pi, \pi]$.

8. Show that $\lim_{x \to \infty} x^n e^{-x} = 0$ for every n.

(5 × 1 = 5)

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. If f and g are each of bounded variation on $[a,b]$, show that their sum, difference and product are also of bounded variations on $[a,b]$. Also show that $V_{f \pm g} \leq V_f + V_g$ and $V_{f \cdot g} \leq AV_f + BV_g$, Where $A = \sup\{|g(x)| : x \in [a,b]\}$ and $B = \sup\{|f(x)| : x \in [a,b]\}$.

10. Establish the additive property of the arc length.

11. Show that $f \in R(\alpha)$ on $[a,b]$ if and only for every $\in > 0$, there exists a partition P such that $U(p,f,\alpha) - L(p,f,\alpha) < t$.

12. Establish integration by parts.

13. State and prove Cauchy criterion for uniform convergence.

14. Show that $\mathscr{C}(X)$, the set of all complex valued, continuous bounded functions on a metric space X is a complete metric space with respect to the norm defined by $\| f \| = \sup_{x \in X} | f(x) |$.

15. Suppose that the series $\sum_{n=0}^{\infty} C_n x^n$ converges for $|x| < R$ and define $f(x) = \sum_{n=0}^{\infty} C_n x^n$ for $|x| < R$.

Show that the series $\sum_{n=0}^{\infty} C_n x^n$ converges uniformly on $[-R + \in, R - \in]$ for any $\in > 0$ and the function f is continuous and differentiable in $(-R, R)$ and also $f'(x) = \sum_{n=1}^{\infty} nC_n x^{n-1}, |x| < R.$

16. Establish the algebraic completeness property of the complex field.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. Establish the additive property of total variation.

18. (a) If $f_1 \in R(\alpha)$ and $f_2 \in R(\alpha)$ on $[a,b]$, show that $f_1 + f_2 \in R(\alpha)$ and $cf \in R(\alpha)$ for every constant c.

(b) State and prove fundamental theorem of calculus.

19. Assume α increases monotonically and $\alpha' \in R$ on $[a,b], f$ be a bounded real function on $[a,b]$.

Show that $f \in R(\alpha)$ iff $f\alpha' \in R$. Also show that $\int_a^b f \, d\alpha = \int_a^b f(x)\alpha'(x)dx.$

20. (a) Suppose k is a compact set and (a) $\{f_n\}$ is a sequence of continuous functions on k ; (b) $\{f_n\}$ converges pointwise to a continuous function f on k ; (c) $f_n(x) \geq f_{n+1}(x)$ for all $x \in k, n = 1, 2, \ldots,$ then show that $f_n \to f$ uniformly on k.

(b) Let α be monotonically increasing on $[a,b]$. Suppose $f_n \in R(\alpha)$ on $[a,b]$ for n = 1, 2, 3, ... and suppose $f_n \to f$ uniformly on $[a,b]$. Show that $f \in R(\alpha)$ on $[a, b]$ and $\int_a^b f \, d\alpha = \lim_{x \to \infty} \int_a^b f_n \, d\alpha.$

21. Suppose $\{f_n\}$ is a sequence of functions, differentiable on $[a,b]$ and such that $\{f_n(x_0)\}$ converges for some x_0 on $[a,b]$. If $\{f_n^{'}\}$ converges uniformly on $[a,b]$, then show that $\{f_n\}$ converges uniformly on $[a, b]$ to a function f and $f^{'}(x) = \lim\limits_{x \to \infty} f_n^{'}(x)$.

22. State and prove Parseval's theorem.

(3 × 5 = 15)