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Reg. No................................

Name....................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, NOVEMBER 2020

Second Semester

Faculty of Science

Branch I (a)—Mathematics

MT 02 C09—PARTIAL DIFFERENTIAL EQUATIONS

(2012—2018 Admissions)

Time : Three Hours | Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has 1 *weight.*

1. Eliminate the constants a and b from the equation $2z = (ax + y)^2 + b$.

2. Eliminate the arbitrary function f from the equation $z = f(x - y)$.

3. Find a complete integral of the equation $pq = 1$.

4. Show that the equations $xp = yq, z(xp + yq) = 2xy$ are compatible.

5. Find a particular integral of the equation $\left(D^2 - D^1\right) z = e^{x+y}$.

6. Deduce the equation $\dfrac{\partial^2 z}{\partial x^2} + 2\dfrac{\partial^2 z}{\partial x \partial y} + \dfrac{\partial^2 z}{\partial y^2} = 0$ to canonical form.

7. Explain exterior Neumann problem.

8. Explain interior Dirichlet's problem.

$(5 \times 1 = 5)$

Turn over

Part B

Answer any **five** *questions.*
Each question has 2 weight.

9. Solve the equation $\left(x^2z - y^3\right)dx + 3xy^2\,dy + x^3dz = 0$ first showing that it is integrable.

10. Find the integral curves of the equation $\dfrac{dx}{x+z} = \dfrac{dy}{y} = \dfrac{dz}{z+y^2}$.

11. Find a complete integral of $\left(p^2 + q^2\right)x = pz$ and deduce the solution which passes through the curve $x = 0, z^2 = 4y$.

12. Solve $z^2 = pqxy$ by Jacobi's method.

13. Verify that the equation $\dfrac{\partial^2 z}{\partial x^2} - \dfrac{\partial^2 z}{\partial y^2} = \dfrac{2z}{x}$ is satisfied by $z = \dfrac{1}{x}\phi(y-x) + \phi'(y-x)$ where ϕ is an arbitrary function.

14. Find a particular integral of the equation $\left(\mathrm{D}^2 - \mathrm{D}^1\right)z = 2y - x^2$.

15. Prove that $r\cos\theta$ and $r^{-2}\cos\theta$ satisfy Laplace's equations, where r, θ, ϕ are spherical polar co-ordinates.

16. Establish a necessary condition for the existence of the solution of the interior Neumann problem.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question has 5 weight.

17. Verify that the equation $z\left(z + y^2\right)dx + z\left(z + x^2\right)dy - xy\left(x + y\right)dz = 0$ is integrable and find its primitive.

18. Find the general integrals of the linear partial differential equation $px(x+y) = qy(x+y) - (x-y)(2x+2y+z)$.

19. Write down and integrate completely the equations for the characteristics of $\left(1+q^2\right)z = px$ expressing x, y, z and p in terms of ϕ, where $q = \tan\phi$ and determine the integral surface which passes through the parabola $x^2 = 2z,\ y = 0$.

20. Find the solution of the equation $z = \frac{1}{2}\left(p^2+q^2\right) + (p-x)(q-y)$ which passes through the x-axis.

21. Deduce the equation $y^2\frac{\partial^2 z}{dx^2} - 2xy\frac{\partial^2 z}{\partial x \partial y} + x^2\frac{\partial^2 z}{\partial y^2} = \frac{y^2}{x}\frac{\partial z}{\partial x} + \frac{x^2}{y}\frac{\partial z}{\partial y}$ to canonical form and hence solve it.

22. Solve the equation $rq^2 - 2pqs + tp^2 = pt - qs$.

(3 × 5 = 15)