Q. P. Code:	2MMPG4

BHARATA MATA COLLEGE, THRIKKAKARA
FIRST INTERNAL EXAMINATION JANUARY 2020

M. Sc. MATHEMATICS SEMESTER II
COMPLEX ANALYSIS

TIME: $1^1/_2$ Hrs **Max. Weight: 15**

Section A
Answer any FOUR Questions, Each question carries 1 weight

1. Define a sequence of complex numbers. When it is said to be a Cauchy sequence?
2. Define uniform convergence of sequence of functions $\{f_n(x)\}$ defined on a set *E*.
3. Define 'analytic function'. Write a necessary and sufficient condition for a function $f(z)$ to be analytic.
4. Define a linear transformation. When it is called a parallel translation, rotation, a homothetic transformation, an inversion?
5. Find the radius of convergence of the power series (i) $\sum_{n=0}^{\infty} n^p z^n$ (ii) $\sum_{n=0}^{\infty} \frac{z^n}{n!}$

Section B
Answer any THREE Questions, Each question carries 2 weight

6. Show that the limit function of a uniformly convergent sequence of continuous functions is itself continuous.
7. Define 'cross ratio' of four points and show that it is invariant under a bilinear transformation. Also show that cross ratio is real if and only if the four points lie on a circle or on a straight line
8. State and prove the symmetry principle.
9. If $f(z)$ is analytic at z_0 and $f'(z_0) \neq 0$, prove that the mapping by $f(z)$ is conformal at z_0.

Section C
Answer any ONE Question, Each question carries 5 weight

10. Describe the spherical representation of complex numbers. Further show that any circle on the sphere corresponds to a circle or a straight line in the z-plane. Also Find the distance $d(z, z')$ between the stereographic projections of z and z'.
11. Prove that
    (i) For every power series $\sum_{0}^{\infty} a_n z^n$, there exists a number *R*, sequence $0 \le R \le \infty$, called the radius of convergence, with the properties that the series converges absolutely for $|z| < R$ and uniformly for $|z| \le \rho$ where $0 \le \rho < R$ and diverges for $|z| > R$.
    (ii) State and prove Abel's Limit Theorem.