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Reg. No...............................

Name..................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, NOVEMBER 2020

Second Semester

Faculty of Science

Branch I (A) : Mathematics

MT 02 C08—ADVANCED COMPLEX ANALYSIS

(2012—2018 Admissions)

Time : Three Hours | Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question carries 1 *weight.*

1. Prove that a convergent sequence is bounded.
2. Define a power series and state Hadamard's formula for the radius of convergence.
3. Define equicontinuous and normal family of functions.
4. State Arzela's theorem.
5. Show that the function $|x|$ is sub-harmonic.
6. State Schwarz-Christoffel formula.
7. Show that sum of the residues of an elliptic function is zero.
8. Define homotopy of two arcs.

(5 × 1 = 5)

Part B

Answer any **five** *questions.*
Each question carries 2 *weight.*

9. State and prove Cauchy Criterion for uniform convergence of a sequence of functions.
10. Establish Jensen's formula.

Turn over

11. Show that $\frac{\pi}{\sin \pi 2} = \frac{\lim}{m \to \infty} \sum_{-m}^{m} (-1)^n \frac{1}{z-n}$.

12. Prove that on entire function of fractional order assumes every finite value infinitely many times.

13. Establish Harnack's inequality.

14. Prove that a continuous function $v(z)$ is sub-harmonic in Ω if and only if it statistics the inequality

$v(z_o) = \frac{1}{2\pi} \int_0^{2\pi} v\left(z_o + re^{i\theta}\right) d\theta$ for every disk $|z - z_o| \leq r$ contained in Ω.

15. Define Weierstrass $\wp$ function. Obtain the differential equation satisfied by $\wp$ function.

16. Explain the terms germs and sheaves associated with analytic continuation.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question carries 5 *weight.*

17. Define Gamma function. Show that $\Gamma(z+1) = z\ \Gamma(z)$. Obtain Legendre's duplication formula.

18. Explain the Taylor and Laurent Series development.

19. (a) For $\sigma = \text{Re}(s) > 1$, show that $\frac{1}{\xi(s)} = \Pi_{n=1}^{\infty} \left(1 - p_n^{-s}\right)$, where p_n's are ascending sequence of primes.

    (b) Prove that the zeta function ξ can be extended to a meromorphic function in the whole plane whose only pole is a simple pole at $s = 1$ with residue 1.

20. State and prove Riemann Mapping Theorem.

21. Prove that there exists a basis (w_1, w_2) such that the ratio $T = w_2 / w_1$ statistics the following conditions :

    (a) $1\,m\,T > 0$.

    (b) $-\frac{1}{2} < \text{Re } T \le \frac{1}{2}$.

    (c) $|T| \ge 1$.

    (d) $\text{Re } T \ge 0$ if $|T| = 1$.

    Also the ratio T is uniquely determined by these conditions and there is a choice of two, four or six corresponding bases.

22. State and prove Mittag-Leffler Theorem.

(3 × 5 = 15)