BHARATAMATA COLLEGE, THRIKKAKARA
FIRST INTERNAL EXAMINATION JAN.2020
M.Sc. DEGREE PROGRAMME- SEMESTER II
MEASURE AND INTEGRATION

Q.P.Code	2MM PG 5

Time: 1 1/2 Hrs.

Part A (*Answer any 4 questions. Each Question carries weight 1)*

1. Define Lebesgue Outer measure and show that Outer measure is Translation Invariant.
2. Let A be the set of irrational numbers in the interval [0,1]. Prove that $m^*(A) = 0$.
3. Show that the translate of a measurable set is measurable.
4. Show that Outer measure is countable subadditivite.
5. If E and F are measurable sets then show that M(E$\cup F$) =m(E) $+ m(F)$.

Part B (*Answer any 3 questions. Each question carries weight 2)*

6. Show that the union of a countable collection of measurable sets is measurable.
7. Show that every interval is measurable.
8. Show that the collection M of measurable sets is a σ- algebra contains the σ- algebra B of Borel sets.
9. State and prove Vitali Theorem.

Part C (*Answer any 1 question. Each question carries 5 weight)*

12. Show that the Outer measure of an interval is its length.

13. (a)Show that the Cantor set C is closed, uncountable and is of measure zero.

(b) Show that the Cantor Lebesgue function φ is an increasing continuous function that maps [0, 1] into [0,1].

(c) Let φ be the Cantor Lebesgue function and define the function ρ on [0,1] by

ρ (x)= $\varphi(x)$ +x for all x in [0,1].

Then show that ρ is strictly increasing continuous function that maps [0,1] onto [0, 2], maps the Cantor set C onto a measurable set of positive measure and maps a measurable set, a subset of the cantor set onto a non-measurable set.