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Reg. No..............................

Name..................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, NOVEMBER 2020

Second Semester

Faculty of Science

Branch I (a) : Mathematics

MT 02 C07—ADVANCED TOPOLOGY

(2012—2018 Admissions)

Time : Three Hours — Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. Define projection functions. Show that projection functions are open.
2. Define productive property of topological spaces. Show that T_1 is a productive property.
3. Let A be a subset of a space X and $f : A \longrightarrow R$ be continuous. Show that any two extensions of f to X agree on $\overline{A}$.
4. Prove that a second countable space is metrisable if and only if it is T_3.
5. Show that a subset A of a space X is closed if and only if limits of nets in A are in A.
6. Let X, Y be sets, $f : X \longrightarrow Y$ a function and $\mathcal{F}$ a filter on X. Show that the family $f(\mathcal{F})$ is a base for a filter on Y.
7. Show that every continuous real valued function on a countably compact space is bounded and attains its extrema.
8. Define a locally compact space. Give examples of spaces which are : (a) Locally compact ; (b) Not locally compact.

(5 × 1 = 5)

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. Prove that if the product is non-empty, then each co-ordinate space is embeddable in it.
10. Prove that a topological product is regular iff each co-ordinate space is regular.

Turn over

11. Show that the evaluation function of a family of functions is one-to-one if and only if that family distinguishes points.

12. Let S : D —> X be a net in a topological space and let $x \in X$. Show that x is a cluster point of S iff there is a subset of S which converges to $x \in X$.

13. Prove that a topological space is compact iff every family of closed subsets of it, which has the finite intersection property, has a non-empty intersection.

14. Prove that a topological space is compact iff every ultra filter in it is convergent.

15. Prove that a first countable, countably compact space is sequentially compact.

16. Assume that X is Hausdorff and locally compact at a point $x \in X$. Show that the family of compact neighbourhoods of x is a local base at x.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. Let A be a closed subset of a normal space X and suppose f : A —> [–1, 1] is a continuous function. Prove that there exists a continuous function F : X —> [–1, 1] such that $F(x) = f(x)$ for all $x \in A$.

18. Prove that a product of spaces is connected if and only if each co-ordinate space is connected.

19. (a) State and prove embedding lemma.

    (b) Prove that a topological space is a Tychonoff space iff it is embeddable into a cube.

20. (a) Prove that a topological space is Hausdorff iff limits of all nets in it are unique.

    (b) Let X, Y be topological spaces, $x \in X$ and f : X —> Y a function. Show that f is continuous at x iff whenever a filter $\mathcal{F}$ converges to $x \in X$, the image filter $f_{\#}(\mathcal{F})$ converges to $f(x)$ in Y.

21. (a) State and prove Tychonoff theorem.

    (b) For a filter $\mathcal{F}$ on a set X show that the following statements are equivalent :

    (i) $\mathcal{F}$ is an ultra filter.

    (ii) For any $A \subset X$, either $A \in \mathcal{F}$ or $X - A \in \mathcal{F}$.

    (iii) For any $A, B \subset X$, $A \cup B \in \mathcal{F}$ iff either $A \in \mathcal{F}$ or $B \in \mathcal{F}$.

22. Prove that one point compactification of a space is Hausdorff iff the space is locally compact and Hausdorff.

(3 × 5 = 15)