Q. P. Code: 2MMPG2

BHARATA MATA COLLEGE, THRIKKAKARA
FIRST INTERNAL EXAMINATION JAN. 2020
M.Sc. MATHEMATICS, SEM.- II
ADVANCED TOPOLOGY

TIME: $1^1/_2$ Hrs. **Max. Weight:15**

Section A
Answer any 4 Questions
Each question carries 1 weight

1. Prove that a compact subset in Hausdorff space is closed.
2. Let A be any subset of a space X and let $f: A \rightarrow R\ be$ continuous.Then any two extensions of f to X agree on $\bar{A}$.
3. Define box and large box.
4. Prove that intersection of any family of boxes is a box.
5. Let$\sum_{n=1}^{\infty} M_n\ be$ a convergent series of non negative real numbers. Suppose $\{f_n\}$ is a sequence of real valued functions on a space X such that for each $x \in X\ and\ n \in N\ |f_n(x)| \leq M_n$. Then the series $\sum_{n=1}^{\infty} f_n$ converges uniformly to a real valued function on X.

Section B
Answer any 3 Questions
Each question carries 2 weight

6. Show that every regular, Lindeloff space is normal.
7. Let X be a Hausdorff space, $x \in X\ and\ F$ be compact subset of X not containing x.Then $\exists$ open sets U, V such that $x \in U$ and $U \cap V = \emptyset$.
8. For any sets Y,I and J , $(Y^I)^J = Y^{I \times J}$ upto a set theoretic equivalence.
9. Let X be a topological space and (Y,d) be a metric space , $\{f_n\}$ is a sequence of functions from $X\ to\ Y$ which converges uniformly to $f: X \rightarrow Y$.Then if each f_n is continuous, so is f.

Section C
Answer any 1 Question
Each question carries 5 weight

10. If a topological space X is normal then it has the property that for every 2 mutually disjoint closed subsets A,B of X , $\exists$ a continuous function $f: X \rightarrow [0,1]$ such that $f(x) = 0\ \forall\ x \in A\ and\ f(x) = 1\ \forall\ x \in B$.
11. State and prove Tieze characterization of normality theorem.